Hw #5 Solutions

5.2#2. See Figure A at the bottom of the page. The volume is given by

$$\begin{aligned} V &= \int_{-1}^{1} A(x)\ dx \\ &= \int_{-1}^{1} \pi(1-x^2)^2\ dx \\ &= 2\int_{0}^{1} \pi(1-2x^2+x^4)\ dx \quad (\because \text{because } (1-x^2)^2 \text{ even function}) \\ &= 2\pi \left| x - 2.\frac{x^3}{3} + \frac{x^5}{5} \right|_0^1 dy \\ &= \frac{16\pi}{15}. \qquad \blacksquare \end{aligned}$$

5.2#6. See Figure B at the bottom of the page. The volume is given by

$$\begin{aligned} V &= \int_{0}^{1} A(y) dy \\ &= \int_{0}^{1} \pi(y-y^2)^2\ dy \\ &= \int_{0}^{1} (y^2-2y^3+y^4)\ dy \\ &= \pi \left| \frac{y^3}{3} - 2.\frac{y^4}{4} + \frac{y^5}{5} \right|_0^1 dy \\ &= \frac{\pi}{30}. \qquad \blacksquare \end{aligned}$$

5.2#10. See Figure C at the bottom of the page. In this case, we have washer with inner radius $x = 2\sqrt{y}$ (this is obtained by solving given function in terms of x) and outer radius 2. The volume is given by

$$\begin{aligned} V &= \int_{0}^{1} \pi\left[(\text{Outer radius})^2 - (\text{Inner radius})^2\right]\ dy \\ &= \pi\int_{0}^{1} [2^2 - (2\sqrt{y})^2)\ dy \\ &= \pi\int_{0}^{1} (4-4y)\ dy = 4\pi\int_{0}^{1} (1-y)\ dy \\ &= 4\pi \left| y - \frac{y^2}{2} \right|_0^1 dy \\ &= 2\pi. \qquad \blacksquare \end{aligned}$$

5.2#18. See Figure D at the bottom of the page. The formula to find volume is

$$V = \int_c^d A(y)dy$$

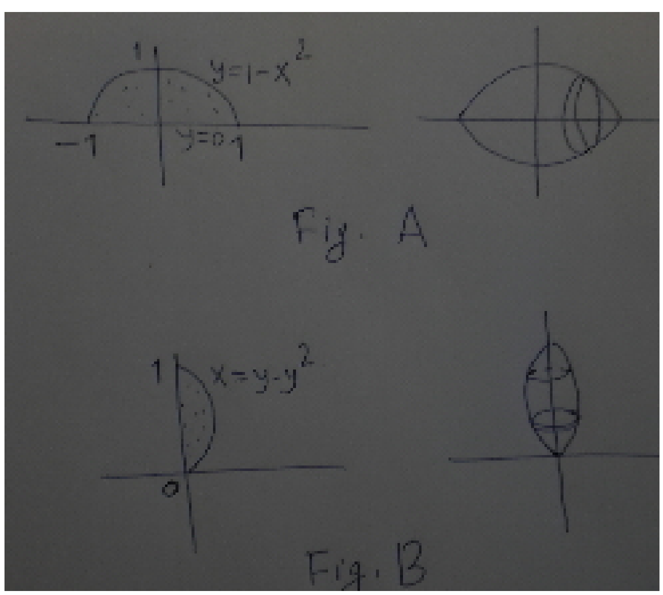
The limits for y are from $y = 0$ to $y = 4$. For $0 \leq y \leq 2$, cross section is annulus with inner radius $2 - 1 = 1$ and outer radius $4 - 1 = 3$, and for $2 \leq y \leq 4$, cross section is annulus with inner radius $y - 1$ and outer radius $4 - 1 = 3$. Therefore, volume is given by

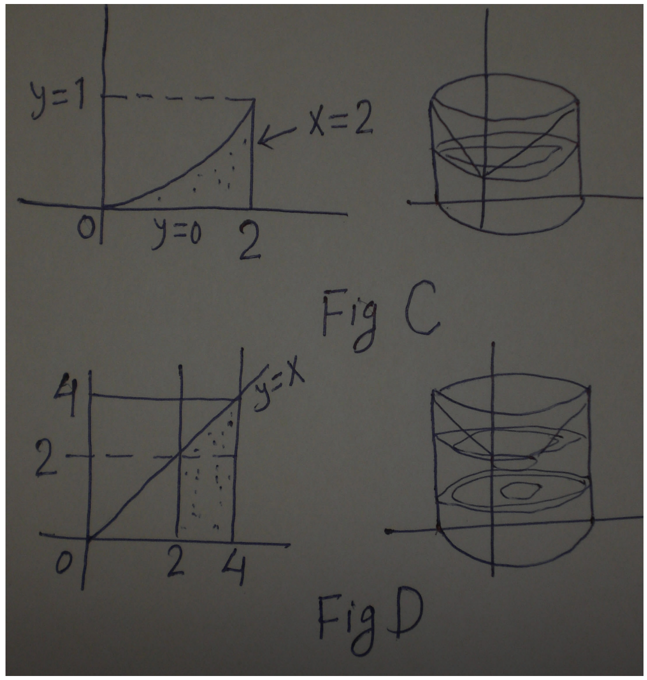
$$\begin{aligned}
V &= \int_0^2 \pi\left[(\text{Outer})^2 - (\text{Inner})^2\right]\, dy + \int_2^4 \pi\left[(\text{Outer})^2 - (\text{Inner})^2\right]\, dy \\
&= \int_0^2 \pi\left[3^2 - 1^2\right]\, dy + \int_2^4 \pi\left[3^2 - (y-1)^2\right]\, dy \\
&= \pi \int_0^2 [9 - 1]\, dy + \pi \int_2^4 \left[9 - (y^2 - 2y + 1)\right]\, dy \\
&= 8\pi \int_0^2 dy + \pi \int_2^4 [8 - y^2 + 2y]\, dy \\
&= 8\pi\, [y]_0^2 + \pi \left| 8y - \frac{y^3}{3} + 2\frac{y^2}{2} \right|_2^4 = \frac{76\pi}{3}. \quad \blacksquare
\end{aligned}$$

5.2#20. OC is the line $x = 0$. The volume rotating $\mathcal{R}_1$ about OC is given by

$$\begin{aligned}
V &= \int_0^1 A(y)\, dy = \int_0^1 \pi(1-y)^2\, dy \\
&= \pi \left| y - \frac{y^2}{2} \right|_0^1 = \frac{2\pi}{3}. \quad \blacksquare
\end{aligned}$$

Figures :




y=1
x=2
0
y=0
2
Fig C
4
2
y=x
0
2
4
Fig D