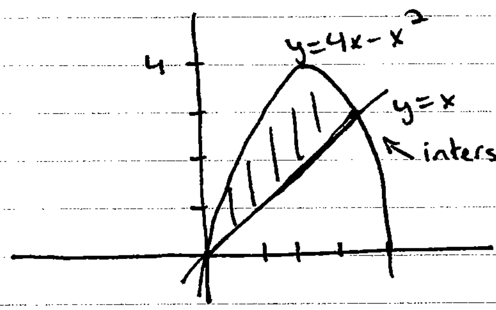
Calc II HW #6

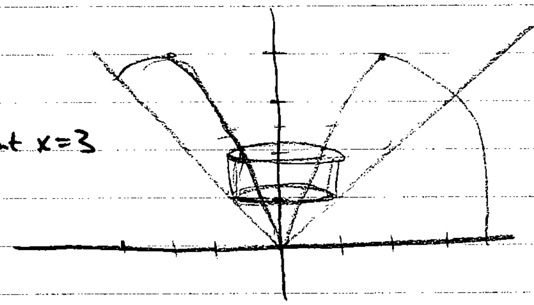
5.3 6, 10, 22(a), 32; 5.5 10

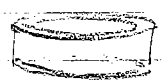
#6 $y = 4x - x^2$, $y = x$ Use shell method to find the volume of the solid generated by rotating the region bounded by these two curves about the y-axis.

First, graph the two curves




Rotate about the y-axis





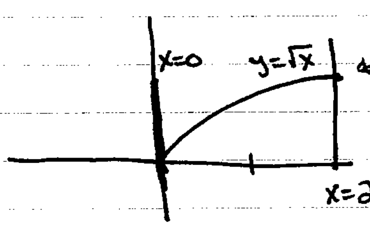
This shell has volume $2\pi r h \Delta r$
where $r = x$, $h = (4x-x^2)-(x)$ = (greater curve) − (lesser curve)
$h = 3x - x^2$

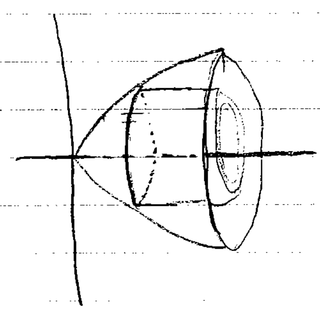
Now integrate with respect to $x$ from 0 to 3: $2\pi \int_0^3 x(3x-x^2)\,dx =$

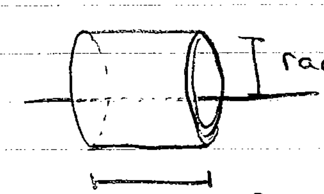
$$2\pi \int_0^3 3x^2 - x^3\,dx$$

$$= 2\pi\left[x^3 - \frac{x^4}{4}\right]_0^3 = 2\pi\left(27 - \frac{81}{4}\right) = 2\pi\left(\frac{27}{4}\right) = \boxed{\frac{27\pi}{2} = 13.5\pi}$$

#10 Use shell method... but now rotate about the x-axis
$y = \sqrt{x}$, $x=0$, $x=2$



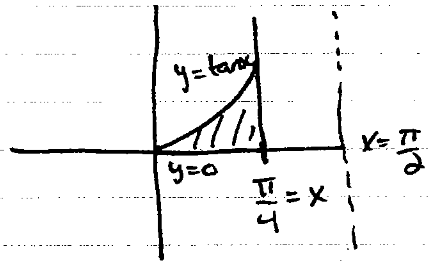


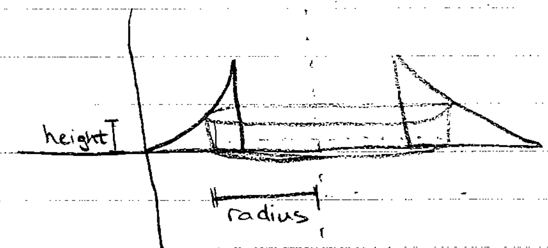

Integrate with respect to $y$ from 0 to $\sqrt{2}$

$$\int_0^{\sqrt{2}} 2\pi y(2-y^2)\,dy = 2\pi\left(y^2 - \frac{y^4}{4}\right)\Big]_0^{\sqrt{2}} = 2\pi\left(2 - \frac{4}{4}\right) = \boxed{2\pi}$$

# 22(a) Set up the integral.

$y = \tan x$, $y = 0$, $x = \pi/4$ about $x = \frac{\pi}{2}$



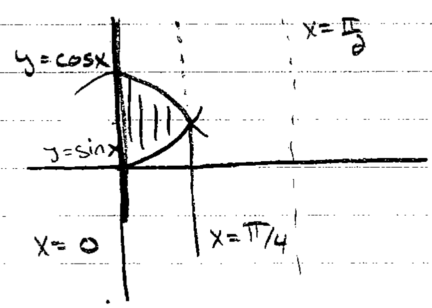

$h = \tan x - 0$ $\quad r = \frac{\pi}{2} - x$

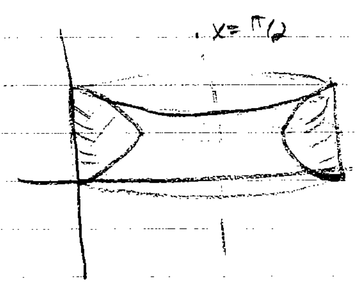
Since we're rotating it about a vertical line, using shell method we integrate with respect to x, from 0 to $\pi/4$

$$\int_0^{\pi/4} 2\pi\left(\frac{\pi}{2} - x\right)(\tan x)\,dx$$

# 32 Describe the solid that $\int_0^{\pi/4} 2\pi(\pi - x)(\cos x - \sin x)\,dx$ represents.

- Since there's a $2\pi$ think shell method.
- Because we're taking the integral with respect to x, we know we're rotating it about a vertical line.
- The radius of each shell is $\pi - x$ which clues us in that the vertical line is $x = \pi$.
- The height of each shell is $\cos x - \sin x$ on the interval 0 to $\pi/4$. So the curves that are bounding the area are $y = \cos x$, $y = \sin x$, $x = 0$, and $x = \pi/4$



5.5 #10 $f(x) = \sqrt{x}$ on $[0,4]$

(a) $f_{ave} = \frac{1}{4-0}\int_0^4 x^{1/2}\,dx = \frac{1}{4} \left. x^{3/2} \cdot \frac{2}{3} \right]_0^4$

$= \frac{1}{6} \left. x^{3/2} \right]_0^4 = \frac{1}{6} \cdot 8 = \boxed{\frac{4}{3}}$

(b) Find c ~~s.t.~~ such that $f_{ave} = f(c)$.

$f_{ave} = 4/3$ & $f(c) = \sqrt{c}$

$\frac{4}{3} = \sqrt{c} \Rightarrow \boxed{c = \frac{16}{9}}$

(c) Sketch the graph of f and a rectangle whose area is the same as the area under the graph of f.