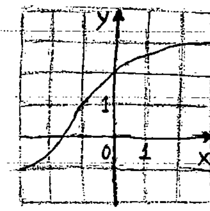
§ 6.1

20. The graph of f is given.

(a) Why is f one-to-one? Because no horizontal line intersects the graph more than once, f is one-to-one.

(b) What are the domain and range of f^{-1}?



domain of f^{-1} = range of f = $[-1, 3]$

range of f^{-1} = domain of f = $[-3, 3]$

(c) What is the value of $f^{-1}(2)$?

By definition 2, $f^{-1}(2) = x \Leftrightarrow f(x) = 2$. As $f(0) = 2$, then $f^{-1}(2) = 0$

(d) Estimate the value for $f^{-1}(0)$

$f(-1.75) \approx 0$. So $f^{-1}(0) \approx -1.75$

26. Find the formula for the inverse of the function.

$y = x^2 - x \quad , x \geq \frac{1}{2}$.

Goal: solve for x. Then, replace x with y^{-1}, y with x.

Meathod 1. Complete the square.

$y = x^2 - x \qquad [-1 \div 2 = -\frac{1}{2} \;;\; (-\frac{1}{2})^2 = \frac{1}{4}]$

$y + \frac{1}{4} = x^2 - x + \frac{1}{4}$

$y + \frac{1}{4} = (x - \frac{1}{2})^2$

$\pm\sqrt{y + \frac{1}{4}} = x - \frac{1}{2}$. As $x \geq \frac{1}{2} \Rightarrow x - \frac{1}{2} \geq 0$. So

$\sqrt{y + \frac{1}{4}} = x - \frac{1}{2}$

$\sqrt{y + \frac{1}{4}} + \frac{1}{2} = x$. So $\underline{y^{-1} = \sqrt{x + \frac{1}{4}} + \frac{1}{2}}$.

Meathod 2. Quadratic formula.

$y = x^2 - x$

$0 = x^2 - x - y$ [Must set = to 0 to use quadratic formula]

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-y)}}{2(1)} = \frac{1 \pm \sqrt{1 + 4y}}{2}$$

As $x \geq \frac{1}{2} \Rightarrow x = \dfrac{1 + \sqrt{1 + 4y}}{2}$. So $\underline{y^{-1} = \dfrac{1 + \sqrt{1 + 4x}}{2}}$

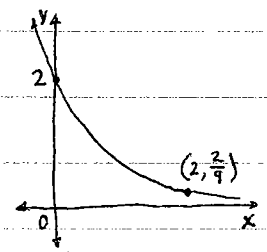
[These two answers are equivalent:

$$\frac{1+\sqrt{1+4x}}{2} = \frac{1}{2} + \frac{\sqrt{1+4x}}{2} = \frac{1}{2} + \frac{\sqrt{1+4x}}{\sqrt{4}} = \frac{1}{2} + \sqrt{\frac{1+4x}{4}}$$

$$= \frac{1}{2} + \sqrt{\frac{1}{4}+x} \quad \checkmark \;]$$

§6.2

18. Find the exponential $f(x) = Ca^x$ whose graph is given.



points: $(0, 2)$, $(2, \frac{2}{9})$

Using $(0, 2)$: $2 = Ca^0$

$2 = C \cdot 1 \Rightarrow C = 2$

Using $(2, \frac{2}{9})$: $\frac{2}{9} = 2a^2 \longrightarrow \frac{1}{9} = a^2 \longrightarrow \pm\frac{1}{3} = a$.

As $a > 0$ [Theorem 2] $\Rightarrow \frac{1}{3} = a$

So, $\underline{f(x) = 2(\frac{1}{3})^x}$

40. Differentiate the function: $f(t) = \sin(e^t) + e^{\sin(t)}$

$$\frac{df}{dt} = \cos(e^t) \cdot \frac{d}{dt}(e^t) + e^{(\sin(t))} \cdot \frac{d}{dt}(\sin(t))$$

$$= \underline{\cos(e^t) \cdot e^t + e^{\sin(t)} \cdot \cos(t)}$$

86. Evaluate the integral: $\int e^x(4+e^x)^5 dx$.

Let $u = 4+e^x$. Then $du = e^x dx$.

$$\int (4+e^x)^5 e^x dx = \int u^5 du = \frac{u^6}{6} + C = \underline{\frac{(4+e^x)}{6} + C}$$

Note to student: ______ The technique of completing the square will be seen in the section of trigonometric integration. If what was done in problem 26 from §6.1 looks unfamiliar, please refresh yourself on this techique before then.