Name: ID #: Section:

Final Exam
Math 2423-010
December 18, 2008

Problem 1: Problem 4:

Problem 2: Problem 5:

Problem 3: Problem 6:

Total:

1(a) A honeybee population starts at 100 and increases at a rate of $n'(t)$ bees per week. What does $100 + \int_0^{18} n'(t)\,dt$ represent?

(b) Give a number x such that $\sin^{-1}(\sin(x)) = x$.

(c) Give a number y such that $\sin^{-1}(\sin(y)) \neq y$.

(d) Find the average value of $\dfrac{(\ln x)^2}{x}$ over the interval $[1, e^3]$.

2(a) Does $\displaystyle\int_0^\infty \frac{x}{x^3+1}\,dx$ converge or diverge? Use a comparison to explain why.

(b) Explain why $\displaystyle\int_3^5 \frac{1}{\sqrt{5-x}}\,dx$ is improper, and evaluate it.

3(a) Find the area of the region bounded by the curves $y = e^x$, $y = \sin x$, $x = 0$, and $x = \pi/2$.

(b) Evaluate $\int_0^2 y^2 \sqrt{1 + y^3}\, dy$.

4(a) Use a trigonometric substitution to find $\displaystyle\int \frac{1}{x^2\sqrt{x^2+4}}\,dx$.

(b) Find $\displaystyle\lim_{x\to 0} \frac{x}{\tan^{-1}(4x)}$.

5(a) Write out the appropriate form of the partial fraction decomposition. Do *not* determine the numerical values of the coefficients. [Hint: be careful.]

(i) $\dfrac{x+5}{(x^2+4)(x-2)^2}$

(ii) $\dfrac{x^2+1}{(x+1)(x^2-1)(x+2)}$

(b) Evaluate $\displaystyle\int_0^2 \frac{x^3}{x-2}\,dx$.

6. Let R be the region bounded by $y = \sin x$, $x = 2\pi$, $x = 3\pi$, and $y = 0$. Use shells to find the volume of the solid obtained by rotating R about the y-axis. [Draw pictures. What is the area of a typical shell?]

Page 7: Some identities

$\sin^2 x + \cos^2 x = 1$
$1 + \tan^2 x = \sec^2 x$
$\sin^2 x = \frac{1}{2}(1 - \cos(2x))$
$\cos^2 x = \frac{1}{2}(1 + \cos(2x))$
$\sin(2x) = 2 \sin x \cos x$
$\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$
$\sin A \cos B = \frac{1}{2}(\sin(A - B) + \sin(A + B))$
$\sin A \sin B = \frac{1}{2}(\cos(A - B) - \cos(A + B))$
$\cos A \cos B = \frac{1}{2}(\cos(A - B) + \cos(A + B))$

Extra Credit Use logarithmic differentiation to find $\frac{dy}{dx}$ when $y = x^{(x^x)}$.