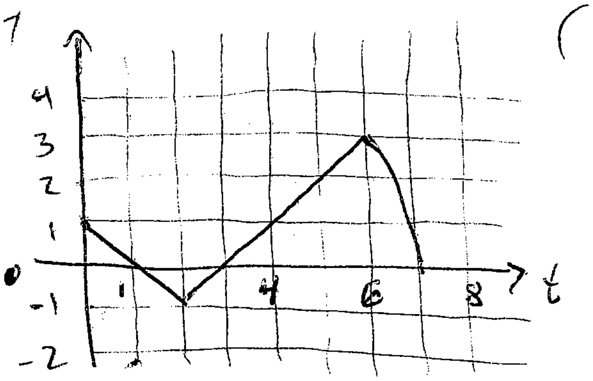
1(a) Let $g(x) = \int_0^x f(t)\,dt$, where f is the function whose graph is shown.

(i) Evaluate $g(x)$ for $x = 1, 3, 5, 6$.
(ii) On what intervals is g increasing? Decreasing?
(iii) Where does the maximum of $g(x)$ occur?
(iv) What is $g'(4)$?



(i) $\boxed{\begin{array}{l} g(1) = \frac{1}{2} \\ g(3) = \frac{-1}{2} \\ g(5) = \frac{3}{2} \\ g(6) = 4 \end{array}}$

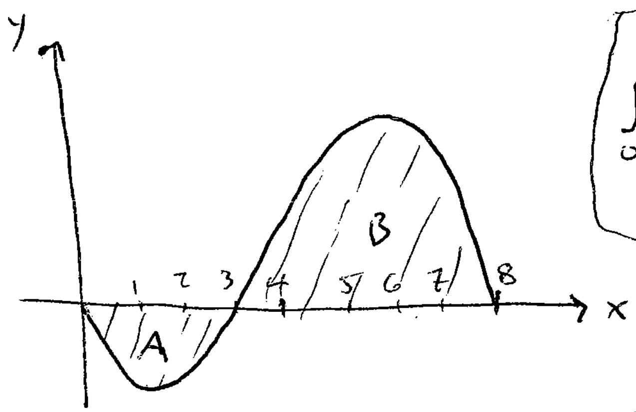
(ii) increasing when $0 < x < 1$ or $3 < x < 7$. decreasing when $1 < x < 3$.

(iii) Maximum occurs at $\boxed{x = 7}$.

(iv) $g'(4) = f(4) = \boxed{1}$.

1(b) Put the following quantities into order, from smallest to largest. Explain your reasoning.

$$\int_0^8 f(x)\,dx \qquad \int_0^3 f(x)\,dx \qquad \int_3^8 f(x)\,dx \qquad \int_4^8 f(x)\,dx$$



$$\boxed{\int_0^3 f(x)dx < \int_0^8 f(x)dx < \int_4^8 f(x)dx < \int_3^8 f(x)dx}$$

Let A, B be the areas indicated (both are positive numbers).
The first integral is $-A$. The second is $B - A$.
The third is slightly less than B, and the fourth is B.

2. Consider the region R enclosed by the curves $y = x(5-x)$ and $y = x$.

(a) Draw the region carefully.

(b) Find the area of this region.

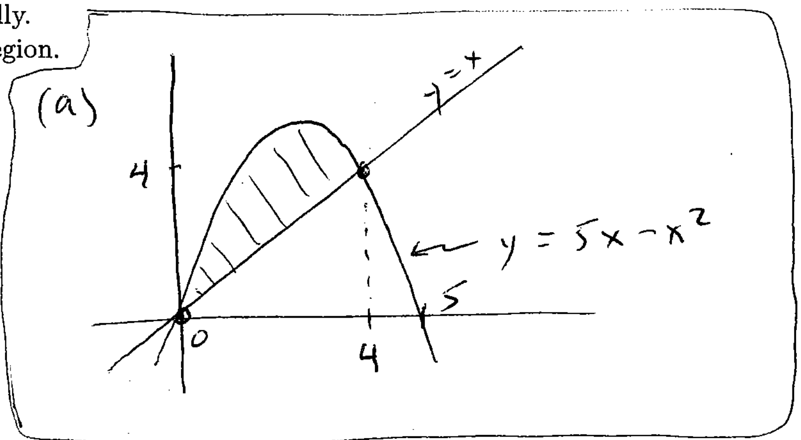
Intersection:

$x = 5x - x^2$

$0 = 4x - x^2$

$= x(4-x)$

$x=0, \ x=4$



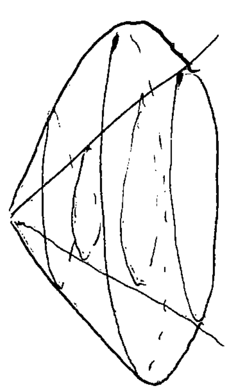
(b) Area is $\int_0^4 ((5x-x^2) - x)\,dx$

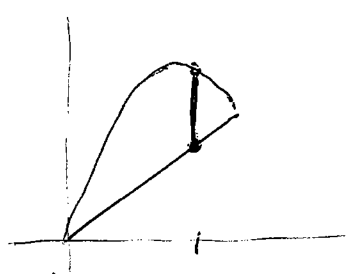
$= \int_0^4 (4x - x^2)\,dx = 2x^2 - \frac{1}{3}x^3 \Big|_0^4$

$= 2(4)^2 - \frac{1}{3}(4)^3 = 32 - \frac{64}{3}$

$= \boxed{32/3}$

(c) Consider the solid of revolution obtained by rotating the region R about the x-axis. Write down a definite integral which represents the volume of this solid.





$A(x) = \pi\left((x(5-x))^2 - x^2\right)$

$\text{Volume} = \int_0^4 A(x)\,dx = \boxed{\int_0^4 \pi\left((x(5-x))^2 - x^2\right)dx}$

3. Evaluate the following:

(a) $\int t\sqrt{1-t^2}\,dt$

Let $u = 1-t^2$

$du = -2t\,dt$

$$\int t\sqrt{1-t^2}\,dt = \int -\frac{1}{2}\sqrt{u}\,du$$

$$= \frac{-1}{2}\cdot\frac{2}{3}u^{3/2} + C$$

$$= \boxed{\frac{-1}{3}(1-t^2)^{3/2} + C}$$

(b) $\int_0^{\pi/8} \sec(2\theta)\tan(2\theta)d\theta$

Let $u = 2\theta$

$du = 2\,d\theta$

$$= \int_0^{\pi/4} \frac{1}{2}\sec(u)\tan(u)\,du$$

$$= \frac{1}{2}\sec(u)\Big|_0^{\pi/4} = \frac{1}{2}\sec(\pi/4) - \frac{1}{2}\sec(0)$$

$$= \frac{1}{2}\left(\frac{2}{\sqrt{2}} - 1\right) = \boxed{\frac{1}{2}(\sqrt{2}-1)}$$

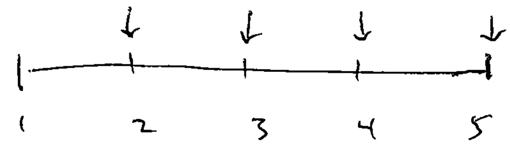
4(a) If $r(t)$ is the rate at which water flows into a reservoir, in gallons per day, what does $\int_0^{100} r(t)\,dt$ represent? Be as specific as you can.

It represents the net increase, in gallons, in the amount of water in the reservoir during the first 100 days.

(b) Write down an antiderivative $F(x)$ of $f(x) = \sin(e^{-x})$ with the property that $F(1) = 0$.

$$\boxed{\int_1^x \sin(e^{-t})\,dt}$$

(c) Write down a Riemann sum for $f(x) = \cos(x)\tan(x)$ over the interval $1 \le x \le 5$ for $n = 4$, using right endpoints. (Either use sigma notation, or write the sum out completely.)



$$\Delta x = \frac{5-1}{4} = 1$$

Riemann sum is

$$\boxed{\cos(2)\tan(2) + \cos(3)\tan(3) + \cos(4)\tan(4) + \cos(5)\tan(5)}$$

or

$$\boxed{\sum_{i=1}^{4} \cos(i+1)\tan(i+1)}$$