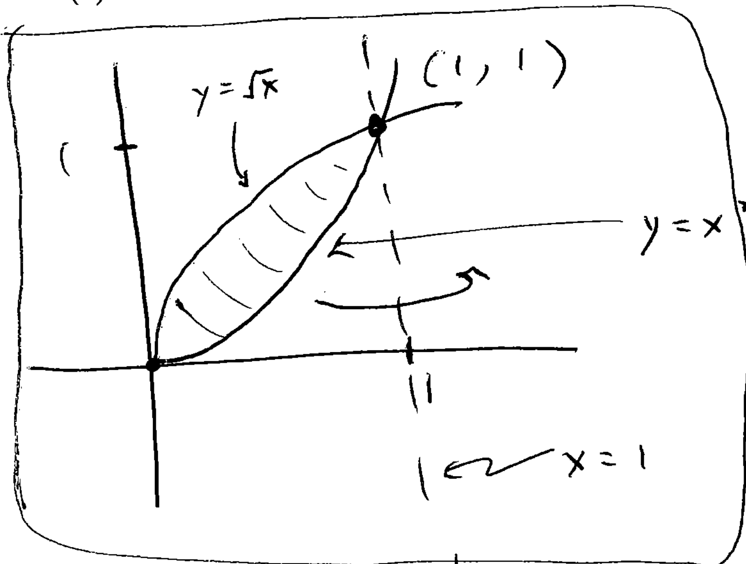
1. Consider the solid obtained by rotating about the line $x = 1$ the region bounded by $y = x^2$ and $y = \sqrt{x}$.

(a) In the xy-plane, draw the region and the axis or rotation.

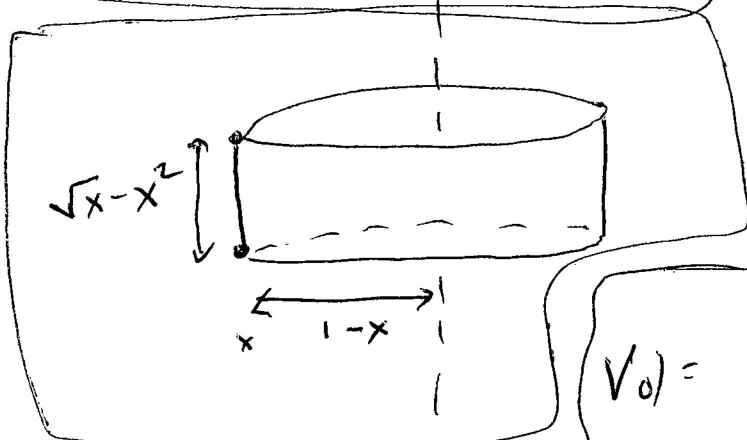
(b) Draw a typical shell, and set up the integral for the volume of the solid using shells.

(c) Find the volume of the solid.

(a)



(b)



Area $A(x) = 2\pi(1-x)(\sqrt{x}-x^2)$

$$\boxed{V_0 = \int_0^1 2\pi(1-x)(\sqrt{x}-x^2)\,dx}$$

(c)

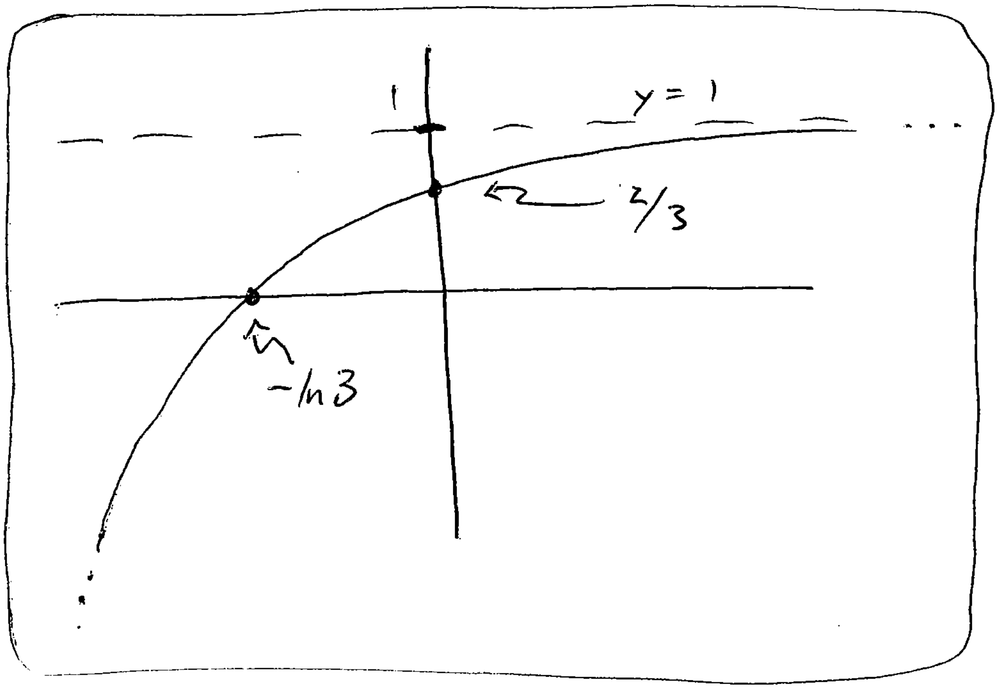
$$= 2\pi \int_0^1 (x^{\frac{1}{2}} - x^{\frac{3}{2}} - x^2 + x^3)\,dx$$

$$= 2\pi \left[\frac{2}{3}x^{3/2} - \frac{2}{5}x^{5/2} - \frac{1}{3}x^3 + \frac{1}{4}x^4 \right]_0^1$$

$$= 2\pi\left(\frac{2}{3} - \frac{2}{5} - \frac{1}{3} + \frac{1}{4}\right) = \boxed{\frac{11}{30}\pi}$$

2. Let $f(x) = 1 - \frac{1}{3}e^{-x}$.
  (a) Draw carefully the graph of $f(x)$. Be sure to label any interesting features.
  (b) Find $\lim_{x\to\infty} f(x)$.
  (c) Find $f^{-1}(x)$ algebraically.
  (d) Determine exactly the intercepts of your graph in (a), if you haven't already.
  (e) Find the domain and range of $f^{-1}(x)$.

(a), (d)



(b)

$$\boxed{\lim_{x\to\infty} f(x) = 1}$$

(horiz. asymptote)

(c) $y = 1 - \frac{1}{3}e^{-x}$, $\quad \frac{1}{3}e^{-x} = 1 - y$

$$e^{-x} = 3 - 3y$$
$$-x = \ln(3-3y)$$
$$x = -\ln(3-3y)$$
$$\boxed{f^{-1}(x) = -\ln(3-3x)}$$

(d) put $x=0$ into f, get $2/3$
put $x=0$ into f^{-1}, get $-\ln 3$

(e) Domain of f^{-1} = range of f = $(-\infty, 1)$

Range of f^{-1} = domain of f = $(-\infty, \infty)$

3. Evaluate the following:

(a) $\int x2^{(x^2)}\,dx$

$u = x^2, \quad du = 2x\,dx$

$= \frac{1}{2}\int 2^u\,du = \frac{1}{2}\cdot\frac{1}{\ln 2}2^u + C$

$= \boxed{\frac{1}{2\ln 2}2^{x^2} + C}$

(b) $\int_0^1 \frac{\sqrt{1+e^{-x}}}{e^x}\,dx$

$u = 1 + e^{-x}, \quad du = -e^{-x}\,dx$

$= \int_2^{1+\frac{1}{e}} -\sqrt{u}\,du = -\frac{2}{3}u^{3/2}\Big|_2^{1+\frac{1}{e}}$

$= \boxed{-\frac{2}{3}\left(1+\frac{1}{e}\right)^{3/2} + \frac{2}{3}(2)^{3/2}}$

(c) $\frac{dy}{dx}$ where $y = 3^{\cos(2x)}$.

$\boxed{\frac{dy}{dx} = \ln 3 \cdot 3^{\cos(2x)}\cdot(-\sin(2x))\cdot 2}$

4(a) If $f''(x) = x^{-2}$, $x > 0$, and $f(1) = 2$, $f'(1) = 0$, find $f(x)$.

$f'(x) = -x^{-1} + C$, $\quad 0 = f'(1) = -1^{-1} + C$

$\Rightarrow C = 1$.

$f'(x) = -x^{-1} + 1$.

$f(x) = -\ln x + x + D$

$2 = f(1) = -\ln(1) + 1 + D = 1 + D$

$\Rightarrow D = 1$

$$\boxed{f(x) = -\ln x + x + 1}$$

(b) If $g(x)$ is the inverse function of $f(x) = 2x + \ln x$, find $g'(2)$.

$$g'(2) = \frac{1}{f'(f^{-1}(2))}$$

and $f'(x) = 2 + \frac{1}{x}$.

$f^{-1}(2) = 1$ b/c $f(1) = 2(1) + \ln(1) = 2$.

$$\text{so } g'(2) = \frac{1}{2 + \frac{1}{1}} = \boxed{\frac{1}{3}}$$