1(a) Find $\lim_{x\to 0} \frac{x^2}{1-\cos x}$.   indeterminate, type $\frac{0}{0}$

$$\overset{L'H}{=} \lim_{x\to 0} \frac{2x}{\sin x}, \quad \text{indeterminate, type } \frac{0}{0}$$

$$\overset{L'H}{=} \lim_{x\to 0} \frac{2}{\cos x} = \boxed{2}$$

(b) Find $\lim_{x\to\infty} x\sin(\pi/x)$.   indeterminate, type $\infty \cdot 0$

$$= \lim_{x\to\infty} \frac{\sin(\pi/x)}{1/x} \quad \text{type } \frac{0}{0}$$

$$\overset{L'H}{=} \lim_{x\to\infty} \frac{\cos(\pi/x)\cdot\pi\cdot\frac{-1}{x^2}}{-1/x^2} = \lim_{x\to\infty} \cos(\pi/x)\cdot\pi$$

$$= \boxed{\pi}$$

(c) Let $g(x) = \cosh(\ln x)$. Find $g'(x)$ and simplify as much as you can.

$$g'(x) = \sinh(\ln x)\cdot\frac{1}{x} = \frac{e^{\ln x} - e^{-\ln x}}{2x}$$

$$= \frac{e^{\ln x} - e^{\ln(x^{-1})}}{2x}$$

$$= \frac{x - x^{-1}}{2x}$$

$$= \boxed{\frac{x^2-1}{2x^2}}$$

2(a) Evaluate $\int_0^{\pi/4} \sec^4 x \tan^{14} x\, dx$. $= \int_0^{\pi/4} \sec^2 x \tan^{14} x \sec^2 x\, dx$

$u = \tan x$, $du = \sec^2 x\, dx$

$$= \int_0^{\pi/4} (1 + \tan^2 x) \tan^{14} x \sec^2 x\, dx = \int_0^1 (1+u^2) u^{14}\, du$$

$$= \int_0^1 (u^{14} + u^{16})\, du$$

$$= \frac{1}{15} u^{15} + \frac{1}{17} u^{17} \Big|_0^1 = \boxed{\frac{1}{15} + \frac{1}{17}}$$

(b) Evaluate $\int te^{-3t}\, dt$. Int. by parts :

$u = t$ $\qquad v = \frac{-1}{3} e^{-3t}$

$du = dt$ $\qquad dv = e^{-3t} dt$

$$\int te^{-3t} dt = \frac{-1}{3} te^{-3t} - \int \frac{-1}{3} e^{-3t} dt$$

$$= \frac{-1}{3} te^{-3t} + \frac{1}{3}\left(\frac{-1}{3} e^{-3t}\right) + C$$

$$= \boxed{\frac{-1}{3} e^{-3t}\left(t + \frac{1}{3}\right) + C}$$

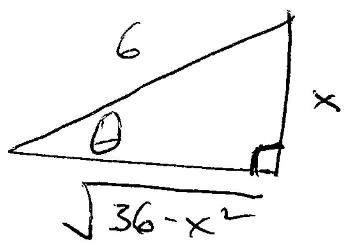
3(a) Write out the form of the partial fraction decomposition of $\dfrac{x^5+11}{(x^2+4)(x^2-4)(x^2-4x)}$. (Do *not* solve for the constants or proceed any further.)

$$Q(x) = (x^2+4)(x+2)(x-2)x(x-4)$$

so

$$\boxed{\frac{x^5+11}{Q(x)} = \frac{Ax+B}{x^2+4} + \frac{C}{x+2} + \frac{D}{x-2} + \frac{E}{x} + \frac{F}{x-4}}$$

(b) If $\sin\theta = x/6$, what is $\tan\theta$?



$$\boxed{\tan\theta = \frac{x}{\sqrt{36-x^2}}}$$

(c) Use a substitution to evaluate $\displaystyle\int \frac{3t^2}{\sqrt{1-t^6}}\,dt$.

$u = t^3$, $du = 3t^2\,dt$

$$\int \frac{3t^2}{\sqrt{1-t^6}}\,dt = \int \frac{1}{\sqrt{1-u^2}}\,du$$

$$= \sin^{-1}(u) + C$$

$$= \boxed{\sin^{-1}(t^3) + C}$$

4(a) Use the substitution $u = \sqrt{x}$ to express the integral $\int_2^3 \frac{\sqrt{x}}{x^2+x}\,dx$ as a rational function, and then evalulate it.

$u = \sqrt{x}$, $u^2 = x$ $\quad dx = 2u\,du$

$\left(\text{or} \quad du = \frac{1}{2\sqrt{x}}dx \Rightarrow 2\sqrt{x}\,du = dx \Rightarrow 2u\,du = dx \right)$

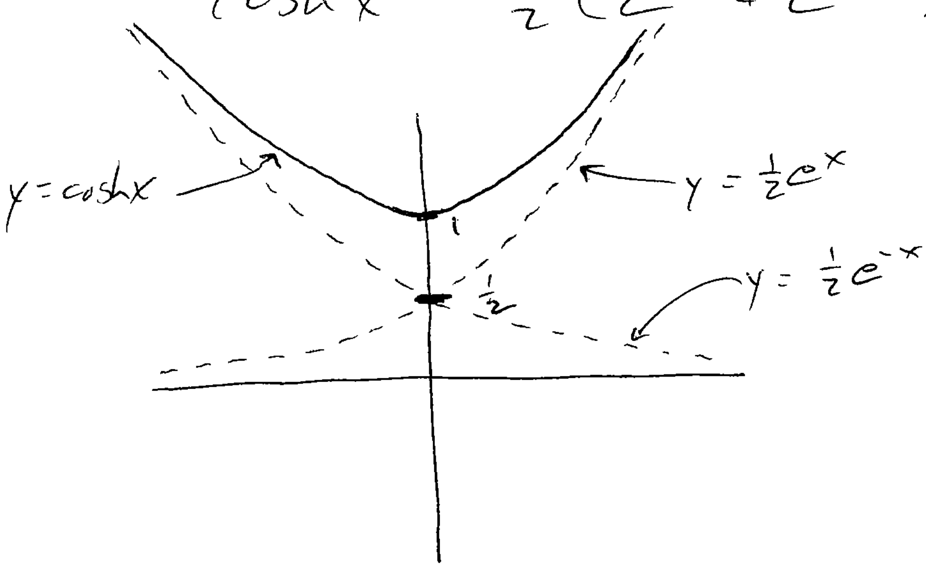
$$\int_{\sqrt{2}}^{\sqrt{3}} \frac{u}{u^4+u^2}\cdot 2u\,du = 2\int_{\sqrt{2}}^{\sqrt{3}} \frac{u^2}{u^2(u^2+1)}du = 2\int_{\sqrt{2}}^{\sqrt{3}} \frac{1}{u^2+1}du$$

$$= 2\tan^{-1}(u)\Big|_{\sqrt{2}}^{\sqrt{3}}$$

$$= \boxed{2\tan^{-1}(\sqrt{3}) - 2\tan^{-1}(\sqrt{2})}$$

(b) Draw carefully the graph of $f(x) = \cosh x$. What are the domain and range?

$\cosh x = \frac{1}{2}(e^x + e^{-x})$



$\boxed{\begin{array}{l}\text{Domain: } (-\infty,\infty) \\ \text{Range: } [1,\infty)\end{array}}$