Homework 2 Solutions
Discrete Math, Spring 2010

Section 1.5

6. Define the following propositions:

R: It rains.
F: It is foggy.
S: The sailing race will be held.
L: The lifesaving demonstration will go on.
T: The trophy will be awarded.

Now we have:

1. $(\neg R \vee \neg F) \to (S \wedge L)$	(premise)
2. $S \to T$	(premise)
3. $\neg T$	(premise)
4. $\neg S$	(modus tollens on 2, 3)
5. $\neg S \vee \neg L$	(addition on 4)
6. $\neg(S \wedge L)$	(deMorgan's law on 5)
7. $\neg(\neg R \vee \neg F)$	(modus tollens on 1, 6)
8. $\neg\neg R \wedge \neg\neg F$	(deMorgan's law on 7)
9. $R \wedge F$	(double negation in 8)
10. R	(simplification on 9)

14. (a) Define the statements:

$C(x)$: x is a student in this class.
$R(x)$: x owns a red convertible.
$T(x)$: x has gotten a speeding ticket.

The argument is:

1. $C(\text{Linda}) \wedge R(\text{Linda})$	(premise)
2. $\forall x(R(x) \to T(x))$	(premise)
3. $R(\text{Linda}) \to T(\text{Linda})$	(universal instantiation on 2)
4. $R(\text{Linda})$	(simplification on 1)
5. $T(\text{Linda})$	(modus ponens on 3, 4)
6. $C(\text{Linda})$	(simplification on 1)
7. $C(\text{Linda}) \wedge T(\text{Linda})$	(conjunction of 5, 6)
8. $\exists x(C(x) \wedge T(x))$	(existential generalization on 7)

(b) Define the statements:

$D(x)$: x has taken a course in discrete mathematics.

$A(x)$: x can take a course in algorithms.

Let the domain for x be the five roomates Melissa, Aaron, Ralph, Veneesha, and Keeshawn.

The argument is:

1. $\forall x D(x)$ (premise)
2. $\forall x(D(x) \to A(x))$ (premise)
3. $D(c)$ for an arbitrary roommate c (universal instantiation on 1)
4. $D(c) \to A(c)$ (universal instantiation on 3)
5. $A(c)$ (modus ponens on 3, 4)
6. $\forall x A(x)$ since c was arbitrary (universal generalization on 5)

(c) Define the statements:

$S(x)$: John Sayles produced the movie x.

$W(x)$: The movie x is wonderful.

$C(x)$: x is a movie about coal miners.

The domain is all movies. Now the argument is:

1. $\forall x(S(x) \to W(x))$ (premise)
2. $\exists x(S(x) \wedge C(x))$ (premise)
3. $S(c) \wedge C(c)$ (existential instantiation on 2)
4. $S(c) \to W(c)$ (universal instantiation on 1)
5. $S(c)$ (simplification of 3)
6. $W(c)$ (modus ponens on 4, 5)
7. $C(c)$ (simplification of 3)
8. $W(c) \wedge C(c)$ (conjunction of 6, 7)
9. $\exists x(W(x) \wedge C(x))$ (existential generalization on 8)

(d) Define the statements:

$C(x)$: x is in this class.

$F(x)$: x has been to France.

$L(x)$: x has visited the Louvre.

Now the argument is:

1. $\exists x(C(x) \wedge F(x))$ (premise)
2. $\forall x(F(x) \to L(x))$ (premise)
3. $C(c) \wedge F(c)$ (existential instantiation on 1)
4. $F(c) \to L(c)$ (universal instantiation on 2)
5. $F(c)$ (simplification of 3)
6. $L(c)$ (modus ponens on 4, 5)
7. $C(c)$ (simplification of 3)

8. $C(c) \wedge L(c)$ (conjunction of 6, 7)
9. $\exists x(C(x) \wedge L(x))$ (existential generalization on 8)

16. **(a)** This is correct; it is basically modus tollens.

(b) Incorrect. Issac's car could be fun to drive while also not being a convertible.

(c) Incorrect. Even if Quincy likes all action movies, he may also like some other movies too. (I don't know whether *Eight Men Out* is an action movie, but this is irrelevant for the argument as it is given.)

(d) This is correct; it is basically modus ponens.

20. **(a)** This argument is invalid. The two premises have the form $\forall x(p(x) \rightarrow q(x))$ and $q(a)$. It does not follow that $p(a)$ is true.

(b) This argument is valid. It is a case of universal instantiation together with modus ponens.

24. The problem is with lines 3 and 5, which do not follow from 2 as claimed. Also, line 7 is not formed correctly, and makes no sense.

28. One possible argument is:

1. $\forall x(P(x) \vee Q(x))$ (premise)
2. $\forall x((\neg P(x) \wedge Q(x)) \rightarrow R(x))$ (premise)
3. $P(c) \vee Q(c)$ for arbitrary c (universal instantiation on 1)
4. $(\neg P(c) \wedge Q(c)) \rightarrow R(c)$ (universal instantiation on 2)
5. $\neg(\neg P(c) \wedge Q(c)) \vee R(c)$ (equivalent to 4)
6. $(\neg\neg P(c) \vee \neg Q(c)) \vee R(c)$ (deMorgan's law on 5)
7. $P(c) \vee \neg Q(c) \vee R(c)$ (double negative in 6)
8. $(P(c) \vee R(c)) \vee \neg Q(c)$ (equivalent to 7)
9. $P(c) \vee (P(c) \vee R(c))$ (resolution on 3, 8)
10. $P(c) \vee R(c)$ (equivalent to 9)
11. $\neg R(c) \rightarrow P(c)$ (equivalent to 10)
12. $\forall x(\neg R(x) \rightarrow P(x))$ since c was arbitrary (universal gen. on 11)

Section 1.6

6. Let a and b be odd numbers. Then there exist integers k, l such that $a = 2k + 1$ and $b = 2l + 1$. We now have

$$ab = (2k + 1)(2l + 1) = 4kl + 2l + 2k + 1 = 2(2kl + l + k) + 1,$$

which is the required form of an odd number. That is, we have shown that $ab = 2m + 1$ for an integer m (namely, $m = 2kl + l + k$). Hence ab is odd.

8. We are told that n is a perfect square, and want to prove that $n + 2$ is not a perfect square. We will assume that $n + 2$ is a perfect square and derive a contradiction; this will prove that our assumption is false (and the theorem we want is true).

So we are assuming that n and $n + 2$ are both perfect squares. This means that there are positive integers a, b such that $a^2 = n$ and $b^2 = n + 2$. Our intuition here is that these numbers are too close together to both be squares. Let's make this precise.

Since $n < n + 2$, taking square roots we get $a < b$. Now consider the number $(a + 1)^2 = a^2 + 2a + 1$. Since a is a positive integer, $a \geq 1$, and so $(a + 1)^2 \geq a^2 + 2 + 1 = (n + 2) + 1 = b^2 + 1$. So we now have

$$a^2 < b^2 < (a + 1)^2.$$

Taking square roots we get $a < b < a + 1$. But this is impossible if a and b are both integers. This contradiction shows that n and $n + 2$ cannot both be squares.

15. We are proving that if $x + y \geq 2$ then $x \geq 1$ or $y \geq 1$.

Suppose the conclusion is false. Then, $x < 1$ and $y < 1$. Adding these, we get $x + y < 1 + 1 = 2$, and therefore the hypothesis "$x + y \geq 2$" is false. This proves the theorem.