Name: ID #:

Exam I

Math 2513-001

February 16, 2009

Problem 1: Problem 2:

Problem 3: Problem 4:

Total:

1. Use a truth table to show that $\neg p \to (q \to r)$ and $q \to (p \vee r)$ are logically equivalent. Don't just give the table; say briefly why the table shows this.

2(a) Let $S(x)$ be the predicate "x is a student," $F(x)$ the predicate "x is a faculty member," and $A(x, y)$ the predicate "x has asked y a question," where the domain consists of all people associated with OU. Use quantifiers to express these statements:

(i) Every student has asked Professor Gross a question.

(ii) There is a faculty member who has never been asked a question by a student.

2(b) Let $T(x, y)$ mean that x likes cuisine y, where the domain for x consists of all students at OU and the domain for y consists of all cuisines. Express each of these statements by a simple English sentence.

(i) $\exists x T(x, \text{Korean}) \wedge \forall x T(x, \text{Mexican})$

(ii) $\exists x \exists z \forall y (x \neq z \wedge (T(x, y) \leftrightarrow T(z, y)))$

3(a) Show that $\forall x(P(x) \to Q(x))$ and $\forall x P(x) \to \forall x Q(x)$ are not logically equivalent.

3(b) Rewrite the following statements so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).

(i) $\neg(\exists x \exists y \neg P(x, y) \wedge \forall x \forall y Q(x, y))$

(ii) $\neg \exists z \forall y \forall x T(x, y, z)$

4. Use the rules of inference (see next page) to show that the premises $\forall x(P(x) \rightarrow (Q(x) \wedge S(x)))$ and $\forall x(P(x) \wedge R(x))$ imply the conclusion $\forall x(R(x) \wedge S(x))$. Justify each step.