Name: ID #:

Exam II
Math 2513-001
March 27, 2009

Problem 1:   Problem 2:

Problem 3:   Problem 4:

Total:

1. Show (either using Venn diagrams or by a direct argument) that if A and B are sets then $(A - B) \cup (A \cap \overline{B}) = A$.

2(a) Let a, b, c be integers. Prove that if $a|b$ and $b|c$ then $a|c$.

2(b) If $A = \{a, b\}$, write out the set (as a list of it elements) $P(A) \times A$. (Recall that P denotes the power set.)

3(a) Find the domain and image of the function that assigns to each positive integer the number of the digits $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$ that do not appear as decimal digits in the integer.

3(b) Determine which of these functions $f \colon \mathbf{Z} \times \mathbf{Z} \to \mathbf{Z}$ is *onto* (and give brief explanations):

(i) $f(m, n) = 2m - n$

(ii) $f(m, n) = m^2 - 4$

(iii) $f(m, n) = |m| - |n|$

4(a) Graph carefully the function $f(x) = 5\lfloor x/5 \rfloor$.

4(b) Find $f^{-1}(\{x \mid 5 < x < 10\})$ and $f^{-1}(\{15, 20\})$.