

Name:

ID #:

Exam III
Math 2513-001
April 27, 2009

Problem 1:

Problem 2:

Problem 3:

Problem 4:

Total:

1. Use induction to prove that $n! < n^n$ for every integer $n \geq 2$:
 - What is the basis step?
 - What is the inductive hypothesis?
 - What do you need to prove in the inductive step?
 - Complete these steps, and the proof.

2(a) Use the Euclidean Algorithm to find $\gcd(9888, 6060)$.

2(b) Find the greatest common divisor and least common multiple of the numbers $3^7 5^3 7^3$ and $2^{11} 3^5 5^9$.

2(c) Explain why $ab = \gcd(a, b) \cdot \text{lcm}(a, b)$ for all positive integers a, b .

3(a) Give the definition for an infinite set S to be *countable*.

Then, suppose that S is countable and F is a finite set with n elements, disjoint from S . Prove that $F \cup S$ is countable.

3(b) What is the coefficient of x^9 in $(2 - x)^{19}$?

4. How many strings of six lowercase letters from the English alphabet contain
the letter a ?
the letters a and b ?
the letters a and b in consecutive positions with a preceding b , with all the letters distinct?