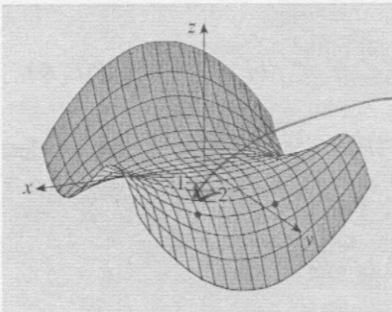
**1(a)** For the function $f(x, y)$ whose graph is shown below, determine whether each of the quantities is zero, positive, or negative.




this is (1, 2)

**(i)** $f_x(1,2)$ positive

**(ii)** $f_y(1,2)$ negative

**(iii)** $f_{xx}(1,2)$ positive

**(iv)** $f_{xy}(1,2)$ positive

**(v)** $f_{yy}(1,2)$ negative

**1(b)** Find the length of the curve given by $\mathbf{r}(t) = \langle \cos(t^2), -t^2, \sin(t^2) \rangle$, where $0 \le t \le \sqrt{2\pi}$.

$$r'(t) = \langle -\sin(t^2) \cdot 2t, -2t, \cos(t^2) \cdot 2t \rangle$$

$$= 2t \langle -\sin(t^2), -1, \cos(t^2) \rangle$$

$$|r'(t)| = 2t \sqrt{\sin^2(t^2) + 1 + \cos^2(t^2)}$$

$$= 2t\sqrt{2} = \sqrt{8}\, t$$

$$\text{length} = \int_0^{\sqrt{2\pi}} |r'(t)|\,dt = \int_0^{\sqrt{2\pi}} \sqrt{8}\, t\,dt = \frac{\sqrt{8}}{2} t^2 \Big|_0^{\sqrt{2\pi}}$$

$$= \frac{\sqrt{8}}{2} \cdot 2\pi = \boxed{\sqrt{8}\,\pi}$$

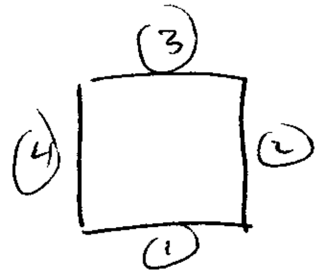
**2.** Find tha absolute maximum and the absolute minimum of the function $f(x, y) = 2x + y - 3xy$ on the region $0 \le x, y \le 1$.

<u>critical points?</u> $f_x = 2 - 3y = 0$, $y = \frac{2}{3}$

$f_y = 1 - 3x = 0$, $x = \frac{1}{3}$

So $\left(\frac{1}{3}, \frac{2}{3}\right)$ is the only critical point.

<u>Boundary?</u>




(1): $f(x,0) = 2x$, $0 \le x \le 1$ linear, so max, min will occur at the endpoints, $(0,0)$, $(1,0)$

(2) $f(1,y) = 2 + y - 3y = 2 - 2y$, $0 \le y \le 1$
linear, so check endpts $(1,0)$, $(1,1)$

(3) $f(x,1) = 2x + 1 - 3x = 1 - x$, $0 \le x \le 1$
linear ... $(0,1)$, $(1,1)$

(4) $f(0,y) = y$, $0 \le y \le 1$, check $(0,0)$, $(0,1)$.

<u>Finally</u> $f\left(\frac{1}{3}, \frac{2}{3}\right) = \frac{4}{3} - \frac{6}{9} = \frac{2}{3}$.

$f(0,0) = 0$ ← abs. min

$f(0,1) = 1$

$f(1,0) = 2$ ← abs. max

$f(1,1) = 0$

$$\boxed{\begin{array}{l} \text{max} = 2, \\ \text{at } (1,0) \\ \text{min} = 0, \\ \text{at } (0,0) \\ \text{and } (1,1) \end{array}}$$

**3(a)** Use the Divergence Theorem to evaluate $\iint_S \mathbf{F}\cdot\mathbf{n}\,dS$ where $\mathbf{F}(x,y,z) = \langle x^2, z^4, e^z\rangle$ and $S$ is the boundary of the box $[0,2]\times[0,3]\times[0,1]$.

$$\operatorname{div}(F) = 2x + e^z$$

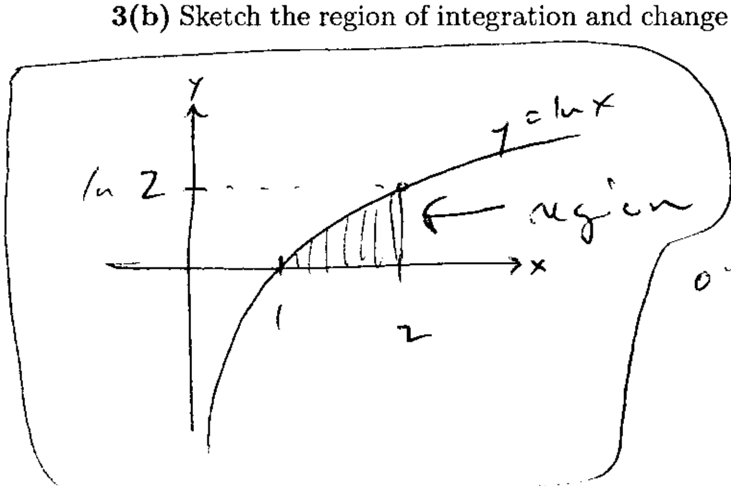
$$\iint_S F\cdot n\, dS = \iiint_E 2x+e^z\, dV = \int_0^1\int_0^3\int_0^2 (2x+e^z)\,dx\,dy\,dz$$

$$= \int_0^1\int_0^3 \left[ x^2+e^z x\Big|_0^2\right] dy\,dz = \int_0^1\int_0^3 (4+2e^z)\,dy\,dz$$

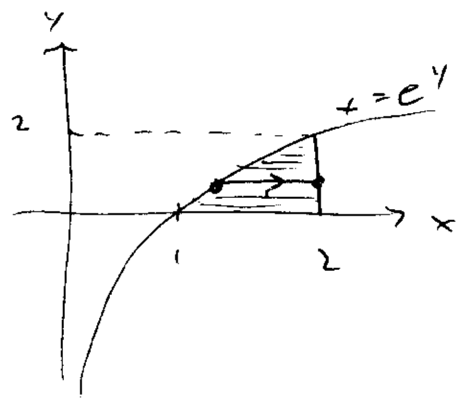
$$= 3\int_0^1 (4+2e^z)\,dz = 3\left[4z+2e^z\Big|_0^1\right]$$

$$= 3(4+2e-2) = \boxed{6+6e}$$

**3(b)** Sketch the region of integration and change the order of integration for $\int_1^2\int_0^{\ln x} f(x,y)\,dy\,dx$.




other order!




$$\boxed{\int_0^{\ln 2}\int_{e^y}^{2} f(x,y)\,dx\,dy}$$

**4.** Use Lagrange Multipliers to find the minimum and maximum values of $f(x, y) = 3x - 2y$ on the circle $\underbrace{x^2 + y^2 = 4}_{g(x,y)}$.

$g(x,y)$, constraint. $\nabla f = \langle 3, -2 \rangle$ $\nabla g = \langle 2x, 2y \rangle$

System:
$$\begin{cases} 3 = \lambda \cdot 2x & (1) \rightarrow \quad 3y = \lambda \cdot 2xy \\ -2 = \lambda \cdot 2y & (2) \rightarrow \quad -2x = \lambda \cdot 2xy \\ x^2 + y^2 = 4 & (3) \end{cases}$$

$$3y = -2x$$
$$y = \frac{-2}{3}x$$

put into (3):

$$x^2 + \left(\frac{-2}{3}x\right)^2 = 4$$
$$x^2 + \frac{4}{9}x^2 = 4$$
$$\frac{13}{9}x^2 = 4 \qquad x^2 = \frac{36}{13}, \qquad x = \pm \frac{6}{\sqrt{13}}$$

then $y = \frac{-2}{3}\left(\pm \frac{6}{\sqrt{13}}\right) = \mp \frac{4}{\sqrt{13}}$ So, the points found by the system are $\left(\frac{6}{\sqrt{13}}, \frac{-4}{\sqrt{13}}\right)$ and $\left(\frac{-6}{\sqrt{13}}, \frac{4}{\sqrt{13}}\right)$.

Evaluate $f\left(\frac{6}{\sqrt{13}}, \frac{-4}{\sqrt{13}}\right) = \frac{18}{\sqrt{13}} + \frac{8}{\sqrt{13}} = \frac{26}{\sqrt{13}}$

$$f\left(\frac{-6}{\sqrt{13}}, \frac{4}{\sqrt{13}}\right) = \frac{-18}{\sqrt{13}} - \frac{8}{\sqrt{13}} = \frac{-26}{\sqrt{13}}$$

absolute maximum is $\frac{26}{\sqrt{13}}$
absolute minimum is $\frac{-26}{\sqrt{13}}$

**5(a)** Let $S$ be the helicoid with parametrization $\mathbf{r}(u,v) = \langle u\cos v, u\sin v, v\rangle$ and $0 \le u \le 1$, $0 \le v \le \pi$. Express $\iint_S x^2 y\, dS$ as an ordinary double integral in $u$ and $v$. Do not solve the integral.

$$r_u = \langle \cos v, \sin v, 0\rangle$$

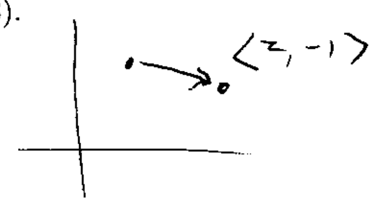
$$r_v = \langle -u\sin v, u\cos v, 1\rangle$$

$$r_u \times r_v = \langle \sin v, -\cos v, u\rangle$$

$$|r_u \times r_v| = \sqrt{\sin^2 v + \cos^2 v + u^2} = \sqrt{1+u^2}.$$

$$\boxed{\iint_S x^2 y\, dS = \int_0^{\pi}\int_0^1 u^2\cos^2 v \cdot u\sin v\sqrt{1+u^2}\, du\, dv}$$

**5(b)** Find the directional derivative of $f(x,y) = x\sqrt{y}$ at the point $(2,4)$, in the direction toward the point $(4,3)$.




$$u = \frac{\langle 2,-1\rangle}{|\langle 2,-1\rangle|} = \frac{\langle 2,-1\rangle}{\sqrt{5}}$$

$$u = \left\langle \frac{2}{\sqrt{5}}, \frac{-1}{\sqrt{5}}\right\rangle, \text{ direction.}$$

$$\nabla f = \left\langle \sqrt{y}, \frac{x}{2\sqrt{y}}\right\rangle$$

$$\nabla f(2,4) = \left\langle 2, \tfrac{1}{2}\right\rangle$$

$$D_u f(2,4) = \nabla f(2,4)\cdot u = \left\langle 2, \tfrac{1}{2}\right\rangle \cdot \left\langle \frac{2}{\sqrt{5}}, \frac{-1}{\sqrt{5}}\right\rangle$$

$$= \frac{4}{\sqrt{5}} + \frac{-1}{2\sqrt{5}} = \boxed{\frac{7}{2\sqrt{5}}}$$

**6(a)** Use the Fundamental Theorem of Line Integrals to evaluate $\int_C 2xe^{-y}\,dx + (2y - x^2e^{-y})\,dy$ where $C$ is any path from $(1,0)$ to $(2,1)$.

$$= \int_C \langle 2xe^{-y}, 2y - x^2e^{-y} \rangle \cdot d\vec{r}$$

need $f$ such that $\nabla f$ is $\langle 2xe^{-y}, 2y - x^2e^{-y} \rangle$

so $f_x = 2xe^{-y}$

$$f(x,y) = x^2e^{-y} + C(y)$$

$$f_y = 2y - x^2e^{-y}$$

$$f(x,y) = y^2 + x^2e^{-y} + D(x)$$

$\Rightarrow$ can take $\underline{f(x,y) = y^2 + x^2e^{-y}}$.

Now $\int_C \nabla f \cdot d\vec{r} = f(2,1) - f(1,0)$

$$= (1 + 4e^{-1}) - (0 + e^0)$$

$$= \boxed{\frac{4}{e}}$$

**6(b)** If $u = x^3y^2 - z^4$, $x = s + 2s^2$, $y = se^s$, and $z = s\sin(s)$, find $\frac{\partial u}{\partial s}$.

$$\begin{array}{ccc} & u & \\ / & | & \backslash \\ x & y & z \\ | & | & | \\ s & s & s \end{array}$$

$$\frac{\partial u}{\partial s} = \frac{\partial u}{\partial x}\frac{dx}{ds} + \frac{\partial u}{\partial y}\frac{dy}{ds} + \frac{\partial u}{\partial z}\frac{dz}{ds}.$$

$$= 3x^2y^2(1+4s) + 2x^3y(se^s + e^s) + (-4z^3)(s\cos(s) + \sin(s))$$

$$= \boxed{3(s+2s^2)^2(se^s)^2(1+4s) + 2(s+2s^2)^3(se^s)(se^s+e^s) - 4(s\sin(s))^3(s\cos(s)+\sin(s))}$$