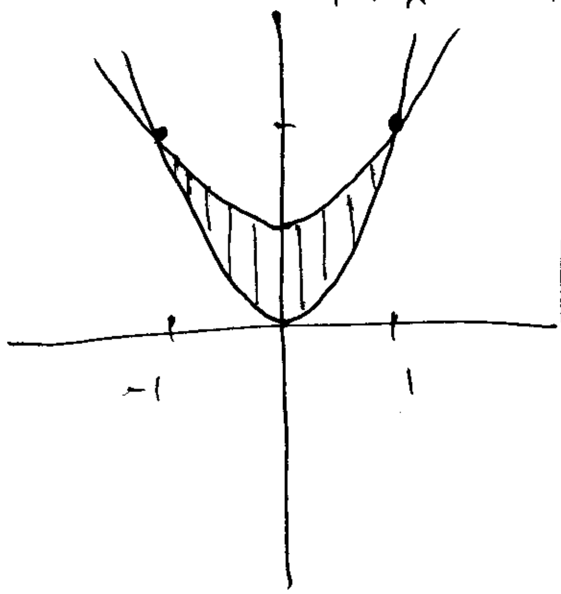
1(a) Set up the integral $\iint_D f(x,y)\,dA$ as an iterated integral, where D is the region between the parabolas $y = 1 + x^2$ and $y = 2x^2$.

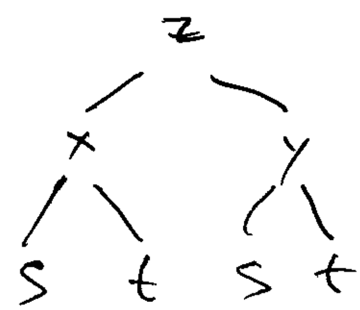
$1 + x^2 = 2x^2, \quad 1 = x^2, \quad x = \pm 1$



$$\boxed{\int_{-1}^{1} \int_{2x^2}^{1+x^2} f(x,y)\,dy\,dx}$$

1(b) Suppose $z = f(x,y)$, where $x = g(s,t)$, $y = h(s,t)$. Suppose also that $g(1,2) = 3$, $g_s(1,2) = -1$, $g_t(1,2) = 4$, $h(1,2) = 6$, $h_s(1,2) = -5$, $h_t(1,2) = 10$, $f_x(3,6) = 7$, and $f_y(3,6) = 8$.

Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ when $s = 1$, $t = 2$.



$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$$

$$\frac{\partial z}{\partial s}(1,2) = f_x(3,6)\,g_s(1,2) + f_y(3,6)\,h_s(1,2)$$

$$= 7\cdot(-1) + 8\cdot(-5) = \boxed{-47}$$

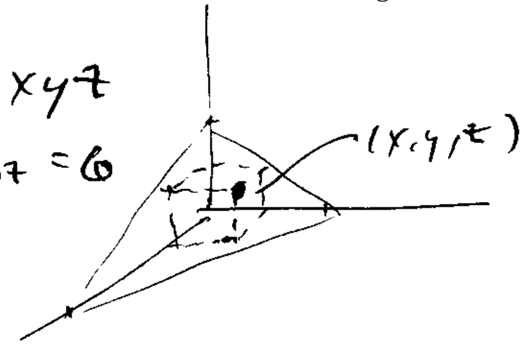
$$\frac{\partial z}{\partial t}(1,2) = f_x(3,6)\,g_t(1,2) + f_y(3,6)\,h_t(1,2)$$

$$= 7\cdot 4 + 8\cdot 10 = \boxed{108}$$

2(a) Consider rectangular boxes in the first octant with three sides in the coordinate planes, and one vertex in the plane $x + 2y + 3z = 6$.

Use Lagrange Multipliers to set up a system of equations whose solution will find the largest volume of such a box. Do *not* solve the system or proceed any further.

Volume of box $= f(x,y,z) = xyz$



Constraint: $g(x,y,z) = x+2y+3z = 6$

Lagrange Multipliers:

$$\begin{cases} \nabla f = \lambda \nabla g \\ g(x,y,z) = k \end{cases}$$

System (in variables x, y, z, λ):

$$\boxed{\begin{aligned} yz &= \lambda \cdot 1 \\ xz &= \lambda \cdot 2 \\ xy &= \lambda \cdot 3 \\ x+2y+3z &= 6 \end{aligned}}$$

2(b) Find the largest directional derivative of $f(x, y) = 4y\sqrt{x}$ at $(4, 1)$, and the direction in which it occurs.

Direction of largest $D_u f$ = direction of ∇f.

Maximum value of $D_u f = |\nabla f|$.

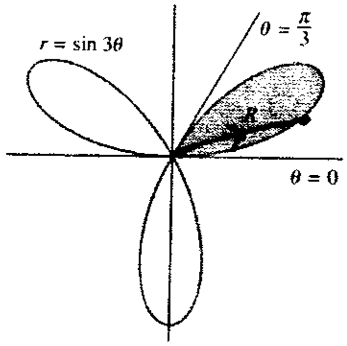
$\nabla f(4,1) = \langle f_x(4,1), f_y(4,1) \rangle \qquad f_x = \frac{2y}{\sqrt{x}}, \quad f_y = 4\sqrt{x}$

$= \langle 1, 8 \rangle$.

So largest $D_u f$ is $|\nabla f| = |\langle 1,8 \rangle| = \boxed{\sqrt{65}}$

in direction $u = \frac{\nabla f}{|\nabla f|} = \boxed{\left\langle \frac{1}{\sqrt{65}}, \frac{8}{\sqrt{65}} \right\rangle}$

3. Use a polar double integral to find the area enclosed by one lobe of the three-petal rose $r = \sin(3\theta)$:



Region is: $\begin{cases} 0 \le \theta \le \pi/3 \\ 0 \le r \le \sin(3\theta) \end{cases}$

$$\text{Area} = \int_0^{\pi/3}\int_0^{\sin(3\theta)} r\,dr\,d\theta$$

$$= \int_0^{\pi/3}\left(\tfrac{1}{2}r^2 \Big|_0^{\sin(3\theta)}\right)d\theta = \int_0^{\pi/3}\tfrac{1}{2}\sin^2(3\theta)\,d\theta$$

$$= \int_0^{\pi/3}\tfrac{1}{4}\left(1-\cos(6\theta)\right)d\theta$$

$$= \int_0^{\pi/3}\tfrac{1}{4}\,d\theta - \int_0^{\pi/3}\tfrac{1}{4}\cos(6\theta)\,d\theta$$

$$= \tfrac{1}{4}\theta\Big|_0^{\pi/3} - \tfrac{1}{4}\cdot\tfrac{1}{6}\sin(6\theta)\Big|_0^{\pi/3}$$

$$= \boxed{\frac{\pi}{12}} \qquad \left(6\cdot\pi/3 = 2\pi\right)$$

4. Find all critical points of the function $f(x, y) = x^3 - 3x + 3xy^2$. Then determine which of these are local maxima, which are local minima, and which are neither.

$f_x = 3x^2 - 3 + 3y^2 = 0$, $\quad f_y = \underline{6xy = 0}$

$x = 0$ or $y = 0$.

If $x = 0$, get $-3 + 3y^2 = 0$

$y^2 = 1$, $y = \pm 1$. $\quad (0, \pm 1)$

If $y = 0$, get $3x^2 - 3 = 0$, $x^2 = 1$, $x = \pm 1$ $\quad (\pm 1, 0)$.

Critical Points: $(0, \pm 1), (\pm 1, 0)$

$f_{xx} = 6x$, $f_{yx} = 6y$, $f_{yy} = 6x$

$D = f_{xx} f_{yy} - f_{yx}^2 = 36x^2 - 36y^2$.

$D(1, 0) = 36 > 0$, $f_{xx}(1, 0) = 6 > 0$

local min. at $(1, 0)$

$D(-1, 0) = 36 > 0$, $f_{xx}(-1, 0) = -6 < 0$

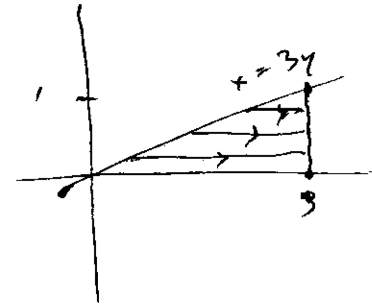
local max. at $(-1, 0)$

$D(0, \pm 1) = -36 < 0$

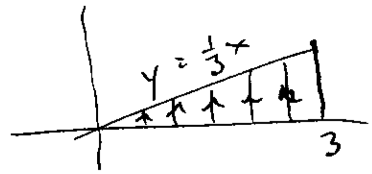
saddle points at $(0, \pm 1)$
i.e. <u>not</u> a local max. or min.

5(a) Evaluate the integral $\int_0^1 \int_{3y}^3 e^{x^2}\, dx dy$ by reversing the order of integration.

region: $0 \le y \le 1,\ 3y \le x \le 3$:



reversing order:



$$\int_0^3 \int_0^{\frac{1}{3}x} e^{x^2}\, dy\, dx$$

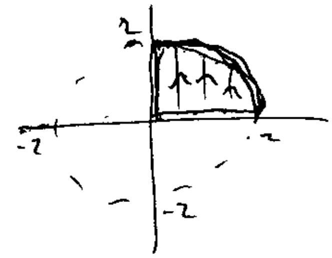
$$= \int_0^3 \left(y e^{x^2} \Big|_0^{\frac{1}{3}x} \right) dx = \int_0^3 \tfrac{1}{3} x e^{x^2}\, dx$$

$u = x^2,\ du = 2x\, dx$

$$= \int_0^9 \tfrac{1}{6} e^u\, du = \boxed{\tfrac{1}{6}(e^9 - 1)}$$

5(b) Convert the integral $\int_0^2 \int_0^{\sqrt{4-x^2}} e^{-x^2-y^2}\, dy dx$ to polar coordinates, but do *not* solve it.

region: $0 \le x \le 2,\ 0 \le y \le \sqrt{4-x^2}$



$$\boxed{\int_0^{\pi/2} \int_0^2 e^{-r^2}\, r\, dr\, d\theta}$$