Take-home Exam II
Algebraic Topology
Official Due Date: November 30, 2006

Advice: Wherever possible, try to use the long exact sequence of a pair or the Excision theorem. You may use problem 27(a) from section 2.1 of Hatcher as well.

1(a) Show that $H_0(X, A) = 0$ if and only if A meets each path component of X.

(b) Show that $H_1(X, A) = 0$ if and only if $H_1(A) \to H_1(X)$ is surjective and each path-component of X contains at most one path-component of A.

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2. Compute the groups $H_n(X, A)$ and $H_n(X, B)$ for X a closed orientable surface of genus two with A and B the circles shown. [What are X/A and X/B?]

See problem 17(b) of section 2.1 of Hatcher for the picture.

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3. Let X be the cone on the 1-skeleton of Δ^3, the union of all line segments joining points in the six edges of Δ^3 to the barycenter of Δ^3. Compute the local homology groups $H_n(X, X - \{x\})$ for all $x \in X$. Using this, identify subsets $A \in X$ with the property that $f(A) = A$ for all homeomorphisms $f\colon X \to X$.

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4. Show that $\widetilde{H}_n(X) \cong \widetilde{H}_{n+1}(SX)$ for all n, where SX is the suspension of X (two copies of the cone CX joined along X). [There is a useful result in section 2.1 of Hatcher.]

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5. Show that $S^1 \times S^1$ and $S^1 \vee S^1 \vee S^2$ have isomorphic homology groups in all dimensions, but their universal covering spaces do not.