Final Exam
Algebraic Topology
December 13, 2006

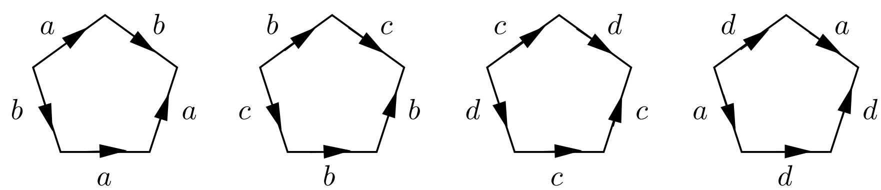
You may apply theorems from the course, but please give the name or statement of the theorem.

1. Compute the homology groups of $S^3 \times S^5$ by using a product cell structure and cellular homology.

2. Find the local homology groups $H_n(X, X - \{x\})$ for various points x in the Möbius band and in the annulus $S^1 \times I$. Then show that these two spaces are not homeomorphic. [Consider their boundaries.]

3. Recall that the *augmentation map* $\varepsilon\colon C_0(X) \to \mathbb{Z}$ take a 0-chain $\sum_i n_i \sigma_i$ to the integer $\sum_i n_i$. Prove that if X is non-empty and path connected then ε induces an isomorphism $H_0(X) \to \mathbb{Z}$.

4. **(a)** Using cellular homology, find the homology groups of the 2-complex X which is defined as follows. It has one 0-cell, four 1-cells (a, b, c, and d), and four 2-cells attached to the 1-skeleton as shown below:



(b) Write down a presentation for the fundamental group of X. [It turns out that this group is infinite, though this is not at all obvious.]

5. **(a)** Given $n > 1$, construct a space X such that $H_1(X)$ is cyclic of order n.

(b) Can you construct a space Y such that $H_2(Y)$ is cyclic of order n?

6. **(a)** Define the *degree* of a map $f\colon S^n \to S^n$.

(b) Prove that the antipodal map is not homotopic to the identity map if n is even. State carefully each of the properties of the degree that you use.