MATH 1473: Mathematics for Critical Thinking

The University of Oklahoma, Dept. of Mathematics

Study Guide and Notes for Chapter 3

Compiled by John Paul Cook

For use in conjunction with the course textbook:

Mathematics: A Practical Odyssey (Sixth Edition)

By David Johnson and Thomas Mowry

The exercises in this packet come from a variety of sources: the course textbook, and supplementary course materials provided by Ms. Christine Tinsley, University of Oklahoma; additionally, some of the exercises are original.
Section 3.2: Basic Terms of Probability

Note: Section 3.1 is an introductory section; it should be read, but we will not go over it in class

Key Terms and Concepts

Experiment, sample space, event, certain event, impossible event, theoretical probability, relative frequency (empirical probability), odds, Law of Large Numbers, face/picture cards

Practice Problems

1) Write down the formulas for (a) the probability of the event \( E \) occurring, and (b) the odds of the event \( E \) occurring:

2) A jar on your desk contains twelve black, eight red, ten yellow, and five green jellybeans. Being the daredevil that you are, you pick a jellybean without looking.

   a. What is the experiment? Sample space?

   b. Find the probability that the jellybean is black.

   c. Find the probability that the jellybean is red or yellow.

   d. Find the probability that the jellybean is not yellow.

   e. Find the odds of picking a green jellybean.

   f. Find the odds of picking a red or black jellybean.
3) You draw one card from a standard deck (no jokers) without looking. Find both the probability and the odds that the card is

a. A black card
b. A queen of spades
c. A card above 4 (aces high)
d. A face card

4) Use the table to answer the following questions:

2000 U.S. Population, in thousands, by age and gender

<table>
<thead>
<tr>
<th>Age</th>
<th>0-4</th>
<th>5-19</th>
<th>20-44</th>
<th>45-64</th>
<th>65-84</th>
<th>85+</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>9,831</td>
<td>31,454</td>
<td>52,294</td>
<td>30,381</td>
<td>13,212</td>
<td>1,240</td>
<td>138,411</td>
</tr>
<tr>
<td>Female</td>
<td>9,387</td>
<td>29,877</td>
<td>51,781</td>
<td>32,059</td>
<td>17,582</td>
<td>3,028</td>
<td>143,713</td>
</tr>
</tbody>
</table>

Source: this table is from the textbook, pg. 147, which obtained the information from the 2000 U.S. Census, U.S. Bureau of the Census

a. Find the probability that in the year 2000, a U.S. resident was female.

b. Find the probability that in the year 2000, a U.S. resident was male and between 20 and 44 years of age.

5) A family has three children. Using b to stand for boy and g to stand for girl, and using ordered triples such as (b,b,g), give the following:

a. The sample space

b. The event $E$ that the family has exactly two daughters

c. The event $F$ that the family has at least two daughters
6) Solve the following problems involving odds and probabilities:
   a. If \( p(E) = \frac{1}{5} \), find \( o(E) \).
   b. If \( o(E) = 3:2 \), find \( p(E) \).

7) Two coins are tossed.
   a. Find the probability that both are heads.
   b. Find the probability that one is heads and one is tails.

Writing and Understanding

8) Explain the difference between theoretical probability and relative frequency.

9) What does the Law of Large Numbers say about the relationship between the two?

10) Give two examples of a case where relative frequency must be used as a means of calculating the probability.

11) What are the advantages of using what we already know about set theory to define the basic formulas of probability?

Section 3.2 Homework: 4, 6, 8, 10, 12, 14, 15, 16, 20, 22, 24, 26, 28, 40, 46, 48, 62, 66
Section 3.3: Basic Rules of Probability

Key Terms and Concepts

Probability Rules, mutually exclusive (disjoint), probability of a union, probability of a complement, odds

Practice Problems

1) Write down the probability rules:

[1]

[2] [4/5]

[3] [6]

2) Determine whether the events E and F are mutually exclusive. Explain your answer.

a. $E$ is the event “it is raining” and $F$ is the event “it is sunny”.

b. $E$ is the event “being single” and $F$ is the event “being married”.

3) A single card is drawn from a standard deck. Use the probability rules to find the probability that the card is a . . .

a. Jack and red

b. Jack or red

c. Not a red jack

d. Under a four
4) Solve the following problems involving odds and probabilities:

a. If $o(E)=5:9$, find $o(E')$.
b. If $p(E)=2/7$, find $o(E)$ and $o(E')$.

g. Under a four or above a nine

h. Not a face card

e. Above a nine

f. Under a four and above a nine

5) For the following questions, use this information: to determine the effect their salespersons have on purchases, a department store polled 700 shoppers regarding whether or not they made a purchase and whether or not they were pleased with the service they received. Of those who made the purchase, 151 were happy with the service, and 133 were not. Of those who made no purchase, 201 were happy with the service and 215 were not. Use the probability rules to find the probability of the event stated:

a. A shopper made a purchase.
b. A shopper was happy with the service received.
c. A shopper made a purchase or was happy with the service.
d. A shopper made no purchase and was happy with the service.
6) Draw a diagram which shows all of the possible outcomes when a pair of dice is rolled.

7) Using the above diagram, find the probability that the sum is as stated when a pair of dice is rolled:
   a. 7
   b. 8 or 10
   c. 8 or 10 or doubles
   d. Even and less than 5
   e. Odd or greater than 7

8) After interviewing randomly selected voters by calling them at home during dinner, Polls-R-Us calculated the following probabilities:

<table>
<thead>
<tr>
<th></th>
<th>Democrats</th>
<th>Republicans</th>
<th>Green Party</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bush</td>
<td>0.08</td>
<td>0.33</td>
<td>0.02</td>
</tr>
<tr>
<td>Kerry</td>
<td>0.38</td>
<td>0.13</td>
<td>0.06</td>
</tr>
</tbody>
</table>

   a. Find the probability that a voter supported Bush or is a Republican.
   b. Find the probability that a voter supported Bush and is a Republican.
   c. Find the probability that a voter didn’t support Bush and is not a Republican.
9) The Video Emporium rents DVDs and videotapes only. They surveyed their 1,167 rental receipts for the last two weeks. Eight-hundred and thirty-two customers rented DVDs, and 692 rented videotapes.

   a. What is the probability that a customer rents DVDs only?

   b. What is the probability that a customer rents DVDs and videotapes?

10) Mary is taking two courses: photography and economics. Student records indicate that the probability of passing photography is .75, that of failing economics is .65, and that of passing at least one of the two courses is .85. Find the probability of the following:

   a. Mary will pass economics.

   b. Mary will pass both courses.

   c. Mary will fail both courses.

   d. Mary will pass exactly one course.
Writing and Understanding

11) Explain why probability rule #3 makes sense.

12) Explain why it is necessary to subtract $p(E \cap F)$ in probability rule #4 (probability of a union). *Hint:* think back to the analogous formula for sets (cardinality of a union).

13) What is the difference between mutually exclusive events and the impossible event?

Section 3.3 Homework: 12, 14, 18, 21, 28, 30, 37, 41, 47, 49, 60, 65, 74
Section 3.4: Combinatorics and Probability

Key Terms and Concepts

No new key terms; review those from section 2.4

Practice Problems

1) A group of thirty people is selected at random. What is the probability that at least two of them will have the same birthday?

2) Find the following probabilities involving lotteries:
   
a. Find the probability of winning first prize for a 6/53 lottery.

b. Find the probability of winning second prize for a 6/53 lottery.

c. Find the probability of winning third prize for a 6/53 lottery.
3) The following problems involve a standard deck of cards (52 cards, no jokers):

   a. Find the probability of being dealt five spades when playing five-card draw.

   b. Find the probability of being dealt a flush when playing five-card draw (use part a).

   c. Find the probability of being dealt a straight flush.

   d. Find the probability of being dealt three aces and two kings.

   e. Find the probability of being dealt a full house.
4) You order twelve burritos to go from a Mexican restaurant, five with hot peppers and seven without. However, wanting to make you play the guessing game, they “forgot” to label them. If you dive right in and pick three burritos at random, find the probability of each of the following events:

   a. All have hot peppers.

   b. Exactly one has hot peppers.

   c. At least one has hot peppers.

5) Two hundred people apply for three jobs. Sixty of the applicants are women.

   a. If three people are selected at random, what is the probability that all are women?

   b. If three people are selected at random, what is the probability that exactly one is a woman?

   c. If three people are selected at random, what is the probability that there are more women selected than men?
Writing and Understanding

6) Is playing the lottery generally a good idea? Why or why not?

7) Why are probabilities for most games of chance calculated using combinations instead of permutations?

8) Explain why, in calculating the probability of a general kind of hand, such as a flush, it is first necessary to calculate the probability of a specific kind of that hand. For instance, before calculating the probability of a flush with any suit, we first calculated the probability of a flush with spades.

Section 3.4 Homework: 2, 5, 9, 12, 17, 18, 19-26, 27abc, 28abcd
Section 3.6: Conditional Probability

Key Terms and Concepts

Conditional probability, product rule, tree diagram

Practice Problems

1) Write the formula for conditional probability.

2) Write the product rule formula.

3) Write the four rules for probability tree diagrams.

4) Consider the following diagram, and use it to find the probabilities below:

   a. \( p(B' \mid A) \)  
   b. \( p(B' \cap A) \)
   c. \( p(B' \mid A') \)
   d. \( p(B' \cap A') \)
5) In a newspaper poll concerning violence on television, 600 were asked, “What is your opinion of the amount of violence on primetime television – is there too much violence on television?” Their responses are indicated in the following table:

<table>
<thead>
<tr>
<th></th>
<th>Yes</th>
<th>No</th>
<th>Don’t Know</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Men</td>
<td>162</td>
<td>95</td>
<td>23</td>
<td>280</td>
</tr>
<tr>
<td>Women</td>
<td>256</td>
<td>45</td>
<td>19</td>
<td>320</td>
</tr>
<tr>
<td>Total</td>
<td>418</td>
<td>140</td>
<td>42</td>
<td>600</td>
</tr>
</tbody>
</table>

Find the following probabilities, and write what each one means:

a. \( p(N) \)  
d. \( p(W | N) \)  
b. \( p(W) \)  
e. \( p(N \cap W) \)  
c. \( p(N | W) \)  

6) Cards are dealt from a standard 52-card deck. Find the following probabilities:

a. The first card is a club

b. The second card is a club, given that the first card was a club

c. The first and second cards are both clubs

d. The first card is a diamond

e. The second card is a spade, given that the first card was a diamond

f. The first card is a diamond and the second card is a spade
7) A die is rolled. Find the probabilities of the given events:
   a. Rolling a 6
   b. Rolling a 6, given that the number rolled is even
   c. Rolling a 6, given that the number rolled is odd
   d. Rolling an even number, given that a 6 was rolled

8) A pair of dice is rolled. Find the probabilities of the given events:
   a. The sum is 12
   b. The sum 12, given that the sum is even
   c. The sum is 12, given that the sum is odd
   d. The sum is even, given that the sum was 12

9) Five cards are dealt from a standard 52 card deck. Find the probabilities of the given events:
   a. All are spades
   b. The last four are spades, given that the first was a spade
10) Three cards are dealt from a standard 52-card deck. Use a tree diagram to find the probabilities of the given events.

   a. Exactly one is a spade

   b. Exactly two are aces

11) For these questions, use the following information: a PC manufacturer buys 38% of its chips from Japan and the rest from the United States. 1.7% of the Japanese chips are defective, and 1.1% of the American chips are defective.

   a. Find the probability that a chip is defective and made in the United States.

   b. Find the probability that a chip is defect-free.

12) Use the table in problem 5 to answer the following questions (this will help with homework problem 64). Let $Y$ be the event “saying yes” and $M$ be the event “being a man”.

   a. $p(Y|M)$

   b. $p(Y|M')$

   c. $p(Y|M)$

   d. $p(Y|M')$
e. Based on the above probabilities, which event is the complement of the event $Y|M$?

**Writing and Understanding**

13) Explain the result to problem 12e above.

14) Which must be true for any events $A$ and $B$? Explain.

$p(A|B)$ is greater than or equal to $p(A)$

$p(A|B)$ is less than or equal to $p(A)$

sometimes $p(A|B)$ is greater than or equal to $p(A)$, and sometimes it is less than or equal to $p(A)$

**Section 3.6 Homework:** 7, 10, 11, 18, 22, 25, 32, 34, 35, 37, 41, 43, 65
Section 3.7: Independence

Key Terms and Concepts

Independent events, dependent events, product rule for independent events

Practice Problems

1) Two events \( E \) and \( F \) are independent if . . .

2) Two events \( E \) and \( F \) are dependent if . . .

3) Rewrite the two definitions above in words.

4) Use probabilities to determine whether the listed events are independent.
   a. If a die is rolled once, \( E \) is the event “getting a 4,” and \( F \) is the event “getting an odd number”

   b. Using the table from problem 5 in Section 3.6, \( Y \) is the event “responding yes to the question on violence in television” and \( W \) is the event “being a woman”

5) A pair of dice is rolled once.
   a. Find the probability of rolling a 6.

   b. Find the probability of rolling a 6, given that the number rolled is even.
c. Are the two events above independent? Why or why not?

d. Are the two events above mutually exclusive? Why or why not?

6) A card is dealt from a full deck (no jokers).

a. Find the probability of being dealt a jack.

b. Find the probability of being dealt a jack, given that you were dealt a card above a seven (aces high).

c. Are the two events above independent? Why or why not?

d. Are the two events above mutually exclusive? Why or why not?

7) To determine what effect salespeople had on purchases, a department store polled 700 shoppers as to whether or not they made a purchase and whether or not they were pleased with the service. Of those who made a purchase, 125 were happy with the service and 111 were not. Of those who made no purchase, 148 were happy with the service, and 316 were not. Are the events “being happy with the service” and “making a purchase” independent? What conclusion can you make?
8) A PC manufacturer buys 38% of its chips from Japan and the rest from the United States. Of the Japanese chips, 1.7% are defective, whereas 1.1% of the US chips are defective. Are the events “defective” and “Japanese-made” independent? What conclusion can you draw?

Writing and Understanding

9) Explain the difference between the terms “independent” and “mutually exclusive”.

10) Explain how the mathematical definition of independence relates to the everyday definition of independence.

Section 3.7 Homework: 10, 12, 13, 15, 18, 20, 23
The next few pages contain review materials to help you review for the Chapter 3 Exam. Please note that the practice exam provides only a means to review for the exam – the actual exam will not be written directly from the practice exam. Use the practice exam only to get a feel for what the exam will be like.

Additionally, a review assignment from the Chapter 3 Review in the textbook (pg. 216) has been listed below, and may be assigned as homework.

The sources of review that you have for the exam include:

** assigned homework / quizzes

** notes / problems worked in class (i.e. from this packet)

** the practice exam

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** Chapter 3 Review Homework: ** 2, 3, 5, 8-12, 14, 15, 17, 20, 22, 24, 27, 29, 30, 43, 48-50, 54
The following questions are multiple-choice questions. Circle the correct answer. Each one is worth 4 points.

2. Which one of the following statements is **false**?

   a. If E and F are mutually exclusive then \( p(E \cup F) = p(E) + p(F) \).

   - b. If \( p(E) = \frac{a}{b} \) then \( o(E) = a:b \).

   c. \( p(\emptyset) = 0 \)

   d. \( p(E) = 1 - p(E') \)

   e. \( 0 \leq p(E) \leq 1 \)

3. When rolling a pair of dice, what is the probability of rolling a sum that is greater than six given you rolled a sum that is even?

   a. 1/4

   b. 3/7

   - c. 1/2

   d. 7/12

   e. None of the above.
4. When rolling a pair of dice, what is the probability of rolling a sum of nine or a sum of ten or a double?

a. $1/6$

b. $1/3$

c. $1/36$

d. $13/36$

e. None of the above.

5. If you are dealt 6 cards from a standard deck of cards, what is the probability of being dealt 3 hearts and 3 spades?

a. 0.12

b. $0.00000079$

c. 0.031

d. 0.004

e. None of the above.
7. When rolling a pair of dice, what are the odds of rolling a sum of nine?

  a. 1:8  
  b. 8:1  
  c. 1:9  
  d. 9:1  
  e. None of the above.

8. If you draw one card from a standard deck of cards, what is the probability of drawing a picture card or a diamond?

  a. 25/52  
  b. 11/15  
  c. 11/26  
  d. 3/52  
  e. None of the above.

Note: the picture cards are J, Q, K, A

9. If you are dealt 5 cards from a standard deck of cards, what is the probability of being dealt any four of a kind?

  a. 0.000005  
  b. 0.0011  
  c. 0.00024  
  d. 0.000018  
  e. None of the above.
10. There are 6 men and 7 women form which you must select a 4 person committee. What is the probability that the committee will consist of 2 or more men?

- a. $\frac{94}{143}$
- b. $\frac{31}{143}$
- c. $\frac{10}{143}$
- d. $\frac{63}{143}$
- e. None of the above.

11. Given $n(S) = 400$, $n(A) = 194$ and $p(A\mid B) = \frac{97}{200}$, what is $p(A \cup B)$?

- a. $\frac{197}{200}$
- b. $\frac{297}{200}$
- c. $\frac{297}{400}$
- d. $\frac{303}{400}$
- e. None of the above.

12. Three coins are tossed. What is the probability of tossing one or more heads?

- a. $\frac{1}{2}$
- b. $\frac{7}{8}$
- c. $\frac{5}{8}$
- d. $\frac{3}{4}$
- e. None of the above.
Use the following information to answer questions 13-16.
A recent survey was conducted to determine which morning show people like. The following table illustrates the results.

<table>
<thead>
<tr>
<th></th>
<th>The Today Show</th>
<th>Good Morning America</th>
<th>The Early Show</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>237</td>
<td>314</td>
<td>198</td>
</tr>
<tr>
<td>Female</td>
<td>301</td>
<td>216</td>
<td>54</td>
</tr>
<tr>
<td></td>
<td>538</td>
<td>530</td>
<td>252</td>
</tr>
</tbody>
</table>

13. What is the probability a person watches Good Morning America?
   a. 53/132
   b. 9/55
   c. 157/660
   d. 108/265
   e. None of the above.

14. What is the probability a person watches The Early Show or they are a male?
   a. 198/749
   b. 21/110
   c. 947/1320
   d. 3/20
   e. None of the above.

15. Are the events “watching The Today Show” and “being a male” independent or dependent?
   a. Independent
   b. Dependent
16. What is the probability a person watches *The Today Show* given they are a female?

a. 301/1320

b. 301/571

\( \text{\checkmark} \)

c. 301/538

d. 538/571

e. None of the above.

Use the following information to answer questions 17-19.
OU's woman basketball team is playing in a weekend tournament. The probability the team wins the first game is 0.69. If the team wins the first game the probability they will win the second game is 0.77. If the team loses the first game the probability they will win the second game is 0.55.

17. What is the probability the team loses the second game given they won the first game?

a. 0.1587

\( \text{\checkmark} \) b. 0.23

c. 0.92

d. 0.1395

e. None of the above.

18. What is the probability the team wins the last game?

a. 0.5313

b. 0.1705

c. 1.32

\( \text{\checkmark} \) d. 0.7018

e. None of the above.
19. What is the probability the team loses both games?

a. 0.45

b. 0.5313

c. 0.76

d. 0.1395

(Answer choice d is circled)
e. None of the above.