MATH 1473: Exam #2

Part I: Definitions and Basic Results

Problem 1: Define the following terms:

(a) with replacement: selection in which items may be selected again

(b) without replacement: selection in which items may not be selected again

(c) subset: a set that is completely contained in another set

(d) cardinality: the number of elements in a set

(e) empty set: the set with no elements

(f) universal set: the set containing all possible elements in question

(g) permutations: various possible outcomes when order matters (w/o replacement)

(h) combinations: various possible outcomes when order does not matter (w/o replacement)

(i) mutually exclusive/disjoint: two sets with nothing in common

Problem 2: Write the following formulas:

(a) cardinality of a union formula: \( n(A \cup B) = n(A) + n(B) - n(A \cap B) \)

(b) cardinality of the complement formula: \( n(A^\prime) = n(U) - n(A) \)

(c) number of subsets of the set \( A \): \( 2^{n(A)} \)

(d) permutations formula: \( nP_r = \frac{n!}{(n-r)!} \)

(e) combinations formula: \( nC_r = \frac{n!}{(n-r)! \cdot r!} \)

(f) distinguishable permutations formula: \( \frac{n!}{x! \cdot y! \cdot z! \cdot \ldots} \)

(g) DeMorgan’s Laws (for sets):

\[ (A \cup B)' = A' \cap B' \quad \text{and} \quad (A \cap B)' = A' \cup B' \]
Part II: Multiple Choice

Use the following information for problems 3, 4: let \( U = \{1, 2, 3, 4, 5, 6, 7\} \), \( A = \{2, 4, 5, 7\} \), \( B = \{3, 5, 6, 7\} \).

**Problem 3:** Find \( A \cup B' \).

a. \( \{1, 2, 4, 5, 7\} \)  
b. \( \{2, 3, 4, 5, 6, 7\} \)  
c. \( \{2, 4\} \)  
d. \( \{1, 2, 4\} \)  
e. None of these.

**Problem 4:** Find \( A' \cap B \).

a. \( \emptyset \)  
b. \( \{1, 2, 3, 4, 6\} \)  
c. \( \{3, 6\} \)  
d. \( \{5, 7\} \)  
e. None of these.

Use the following information for problems 5, 6: let \( n(U)=700 \), \( n(A)=219 \), \( n(B)=302 \), and \( n(A \cap B)=52 \).

**Problem 5:** Find \( n(A') \).

a. 83  
b. 219  
c. 469  
d. 481  
e. None of these.

**Problem 6:** Find \( n(A \cup B) \).

a. 179  
b. 469  
c. 521  
d. 648  
e. None of these.

**Problem 7:** If \( X \cup Y = Y \), what must be true concerning sets \( X \) and \( Y \)?

a. \( X = Y \)  
b. \( X \cap Y = \emptyset \)  
c. \( Y \subseteq X \)  
d. \( X \subseteq Y \)  
e. None of these.
Problem 8: Let $F = \{ x \mid x$ is a month of the year $\}$. Which of the following statements is false?
   a. $F$ has 4,096 subsets
   b. $\text{August} \notin F$
   c. $\{\text{April, July, November}\} \subseteq F$
   d. $n(F) = 12$
   e. None of these.

Use the following information for problems 9, 10: A survey was taken asking people “What is your favorite soft drink?” The following information was obtained: 500 said Coca-Cola, 595 said Pepsi, 520 said Dr. Pepper, 245 liked all three, 90 did not drink soda, 275 said Coca-Cola and Pepsi, 325 said Coca-Cola and Dr. Pepper, and 155 said only Dr. Pepper.

Problem 9: How many like only Pepsi?
   a. 280
   b. 295
   c. 275
   d. 595
   e. None of these.

Problem 10: What was the total number of respondents?
   a. 1,615
   b. 975
   c. 1,065
   d. 1,370
   e. None of these.

Problem 11: $(A' \cap B)' =$
   a. $A \cap B'$
   b. $A' \cap B$
   c. $A' \cup B$
   d. $A \cup B'$
   e. None of these.

Problem 12: A die is rolled, a coin is tossed, and a letter is chosen. How many different outcomes are possible?
   a. 312
   b. 34
   c. 68
   d. 156
   e. None of these.
Problem 13: A group of seven girls and five boys must select a team of three people. How many teams are possible if the team must consist of more girls than boys?

a. 35
b. 140
c. 105
d. 70
e. None of these.

Problem 14: How many cards in a standard deck are a red card or a jack?

a. 2
b. 26
c. 28
d. 30
e. None of these.

Problem 15: Fifty students must select a student government. The student government must consist of a president, vice-president, secretary, and treasurer. In how many ways can they select the committee?

a. 5,527,200
b. 230,300
c. 6,250,000
d. 200
e. None of these.

Problem 16: How many five-card poker hands consisting of exactly two jacks are possible?

a. 17,296
b. 117,600
c. 19,600
d. 103,776
e. None of these.

Problem 17: In how many ways can you be dealt a flush with any suit except clubs in five-card poker?

a. 1,287
b. 3,861
c. 5,148
d. 154,440
e. None of these.
Problem 18: A group of eleven seniors, seven juniors, six sophomores, and four freshmen must select a committee of five. How many committees are possible if the committee must contain exactly three juniors?

a. 35  
b. 210  
c. 7,350  
d. 13,230  
e. None of these.

\[
\binom{7}{3} \times \binom{11+6+4}{2} = 35 \times 210 = 7350
\]

2 categories: juniors and non-juniors

Problem 19 (Bonus): Suppose that \( n(P \cap Q) = 0 \). Which of the following is true concerning the sets \( P \) and \( Q \)?

I. \( P \cap Q = \emptyset \)  ✔
II. \( n(P \cup Q) = n(P) + n(Q) \)
III. \( P \subseteq Q \)  ✗

a. I only  
b. II only  
c. III only  
d. I, II only  
e. I, II, and III

Part III: Free Response

Problem 21: Prove the following identity using Venn diagrams:

\[ A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \]