Part I: Definitions and basic results [1 pt each]

Problem 1: Define the following terms:
   a. combinations:

   b. permutations:

   c. without replacement:

   d. with replacement:

   e. subset:

   f. cardinal number/cardinality:

   g. mutually exclusive/disjoint:

   h. equal sets:

   i. intersection:

   j. union:

   k. complement:

   l. universal set:

   m. empty set:

   n. improper subset:

   o. proper subset:
Problem 2: Write the following formulas:

a. cardinal number formula for the complement of a set: \( n(A') = \)

b. cardinal number formula for the union of sets: \( n(A \cup B) = \)

c. number of subsets of the set \( A \):

d. formula for \( n \) factorial: \( n! = \)

e. combination formula: \( {}_nC_r = \)

f. permutation formula: \( {}_nP_r = \)

g. distinguishable permutations formula:

h. DeMorgan’s Laws (for sets):

\[ (A \cup B)' = \]

\[ (A \cap B)' = \]
Part 2: Multiple Choice [4 pts each]

**Problem 3**: Let \( U = \{1, 2, 3, 4, 5, 6, 7\} \), \( A = \{2, 4, 5, 7\} \), and \( B = \{3, 5, 6, 7\} \). Find \((A \cup B)'\).

a. \{5, 7\}  
b. \{2, 3, 4, 5, 6, 7\}  
c. \{1\}  
d. \{1, 2\}  
e. None of these.

**Problem 4**: Using the sets \( U \), \( A \), and \( B \) as in Problem 3, find \( A \cap B \).

a. \{5, 7\}  
b. \{2, 3, 4, 5, 6, 7\}  
c. \{1\}  
d. \{1, 2\}  
e. None of these.

**Problem 5**: Find the number of distinguishable permutations in the word COMMITTEE.

a. 45,360  
b. 90,720  
c. 181,440  
d. 362,880  
e. None of these.

**Problem 6**: A group of 7 girls and 5 boys must select a team of 3 people. How many teams are possible if the team must consist of more boys than girls?

a. 80  
b. 70  
c. 105  
d. 700  
e. None of these.
**Problem 7:** An IM football league has 10 teams. If every team must play every other team once in the first round of league play, how many games must be scheduled?

a. 5  
b. 10  
c. 45  
d. 90  
e. None of these.

**Problem 8:** Professor Redrum’s film class conducted a survey of people’s movie-watching habits. He had his students ask people to check the appropriate choices:

I watched a movie in a theatre during the past month.

I watched a movie on DVD during the past month

After the professor had collected the forms and tabulated the results, he told the class that 388 people checked the theatre box, 495 checked the DVD box, 281 checked both boxes, and 98 of the forms were left blank. How many people surveyed watched a movie in a theatre or on a DVD during the past month?

a. 107  
b. 214  
c. 602  
d. 700  
e. None of these.

**Problem 9:** In general, if $X \cup Y = Y$, what must be true concerning sets $X$ and $Y$?

a. $X$ and $Y$ are disjoint.  
b. $X = Y$  
c. $Y \subset X$  
d. $X \subset Y$  
e. None of these.

**Problem 10:** Suppose $V = \{\text{Bradford, Iglesias, Gresham, Murray, Broyles}\}$. How many subsets does $V$ have?

a. 5  
b. 10  
c. 25  
d. 32  
e. None of these.
Problem 11: How many five card poker hands containing exactly two jacks are possible?
   a. 17,296
   b. 117,600
   c. 103,776
   d. 19,600
   e. None of these.

Problem 12: Suppose that \( n(P \cap Q) = 0 \). Which of the following is true concerning the sets \( P \) and \( Q \)?
   I. \( P \cap Q = \emptyset \)
   II. \( n(P \cup Q) = n(P) + n(Q) \)
   III. \( P \subseteq Q \)
   a. I only
   b. II only
   c. III only
   d. I, II only
   e. I, II, and III

Problem 13: 15 seniors, 10 juniors, 6 sophomores, and 5 freshmen must select a committee of 4 that consists of 2 upperclassmen (juniors or seniors), and 2 underclassmen (sophomores or freshmen). How many different ways can the committee be selected?
   a. 16,500
   b. 1,413,720
   c. 710
   d. 4,500
   e. None of the statements are false.

Problem 14: 50 students must select a student government. The student government must consist of a president, vice president, secretary, and treasurer. How many different ways can the committee be selected?
   a. 230,300
   b. 5,527,200
   c. 6,250,000
   d. 200
   e. None of these.
Problem 15: Given \( n(U) = 700 \), \( n(A) = 219 \), \( n(A \cap B) = 52 \), and \( n(B) = 302 \), what is \( n(A') \)?

a. 167  
b. 250  
c. 231  
d. 481  
e. None of these.

Problem 16: All students receive an ID number. The ID number must consist of four nonzero repeatable digits followed by two letters that can not be repeated. How many different ID numbers are possible?

a. 3,407,040  
b. 4,264,650  
c. 2,044,224  
d. 4,435,236  
e. None of these.

Problem 17: How many five-card poker hands consist of all the same suit?

a. 13  
b. 1,287  
c. 5,148  
d. 16,731  
e. None of these.

Problem 18: How many cards in a standard deck are are a queen and a club?:

a. 0  
b. 16  
c. 1  
d. 17  
e. None of these.

Problem 19: A survey was taken asking people “what is your favorite soft drink?” The following information was obtained: 500 said Coca-Cola, 595 said Pepsi, 520 said Dr. Pepper, 245 liked all three, 90 did not drink soda, 275 said Coca-Cola and Pepsi, 325 said Coca-Cola and Dr. Pepper, and 155 said only Dr. Pepper. How many like both Pepsi and Dr. Pepper?

a. 155  
b. 245  
c. 40  
d. 285  
e. None of these.
**Problem 20:** Using the same scenario as in Problem 15, determine how many people liked only Coca-Cola.

a. 145  
b. 355  
c. 30  
d. 80  
e. None of these.

**Problem 21:** When trying to pick a dragon, centaur, and unicorn for his new petting zoo, Elliott narrowed his choices to 3 dragons, 7 centaurs, and 4 unicorns. In how many different ways could Elliott choose his three new animals?

a. 14  
b. 28  
c. 56  
d. 84  
e. None of these.

**Problem 22:** Given the set \{b, l, a, k, e\}, how many different two element subsets does this set have?

a. 25  
b. 10  
c. 32  
d. 120  
e. None of these.

**Problem 23:** Use Venn Diagrams to determine which of the following sets \((A \cup B) \cap (A \cup C)\) is equal to:

a. \(A \cup (B \cap C)\)  
b. \(A \cup B \cup C\)  
c. \((A \cup B) \cap C\)  
d. \((A \cup C) \cap B\)  
e. None of these.