Practice Exam III

Instructions.

1. Show your work clearly. The method or reasoning used to obtain an answer is worth more than the answer itself.

2. Attempt every question. Even if you cannot completely answer a problem, you may receive partial credit for what you do know.

3. You may not use calculators, books, or notes of any kind.

Q1] (a) \( \int \sin^2 \theta \cos^2 \theta \, d\theta \)
(b) \( \int \frac{x^3}{\sqrt{x^2 + 4}} \, dx \)

Q2] (a) Write the forms of the partial fraction decomposition for the following functions. Do not calculate the unknowns. (Hint: Be careful.)
(i) \( \frac{x+1}{x(x-1)(x^2+6)} \), (ii) \( \frac{x^2}{(x^2-1)(x+1)} \), (iii) \( \frac{x^2}{x^2+4x+3} \).
(b) Evaluate \( \int x^2 \ln x \, dx \)

Q3] (a) Use a comparison to determine whether the following improper integrals converge or diverge:
(i) \( \int_1^\infty \frac{\sin^2 x}{x^2 + x} \, dx \)
(ii) \( \int_0^1 \frac{6 + e^{-x}}{x} \, dx \)
(b) Evaluate the integral:
\( \int_0^\infty xe^{-x} \, dx \)

Q4] Evaluate the integrals.
(a) \( \int \sin x \tan x \, dx \)
(b) \( \int \frac{2x + 4}{x^3 - 5x^2 + 4x} \, dx \)
Q5] (a) Find the length of the curve \( y = 2x^\frac{3}{2} + 7 \) on the interval \( 0 \leq x \leq 3 \).
(b) Use trigonometric substitution to evaluate \( \int x^2(1 - 4x^2)^{\frac{3}{2}} \, dx \).

The exam will cover specifically Sections 8.1-8.5, 8.8, and 9.1. You should be prepared for questions and problems related to the following topics. (With respect to theorems, know what the theorem says and how to use it. You will not be asked to prove any theorems.)

1. Integration by parts: Know the formula and how to use it
2. Partial fractions decomposition and integration. Do not worry about “rationalizing substitutions.”
3. Integration of trigonometric functions of the form in 8.2.
4. Integration by trigonometric substitution
5. Be able to identify which integration techniques to use on a particular integral.
6. Improper integrals of both types: how to identify them and how to integrate them. (Note: This might require simple applications of L'Hospital’s Rule.)
7. Be able to use the Comparison Theorem to decide whether a particular improper integral is convergent or divergent.
8. Arc length: Know the formula.
9. While it won’t be specifically covered, you should review the Substitution Rule. Its use may be necessary on integrals from any of the sections on the exam.

The exam will have five questions worth 20 points each, although questions may have multiple parts. There may also be one or two extra credit problems.

I will provide you with the following trigonometric identities:

\[
\begin{align*}
\sin^2 x + \cos^2 x &= 1 \\
\tan^2 x + 1 &= \sec^2 x \\
\sin^2 x &= \frac{1}{2}(1 - \cos 2x) \\
\cos^2 x &= \frac{1}{2}(1 + \cos 2x) \\
\sin 2x &= 2 \sin x \cos x
\end{align*}
\]