Review Problems for the Final

math 2423-001

1. Find the limit

   a) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\sqrt{1+\frac{3i}{n}}$;

   b) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^3}{n^4}$;

   c) $\lim_{x\to\pi} \frac{e^{\sin x}-1}{x-\pi}$;

   d) $\lim_{x\to 0} \frac{x+\tan x}{x-\tan x}$;

   e) $\lim_{x\to 0}(\sin x)^{\tan x}$.

2. Find the area of the region bounded by the curves

   a) $y = \cos x$, $y = \sin 2x$, $x = 0$, $x = \pi/2$;

   b) $y^2 = x$, $x - 2y = 3$;

   c) $y = 5\ln x$, $y = x\ln x$.

3. Find the number b such that the line $y = b$ divides the region bounded by the curves $y = x^2$ and $y = 4$ into two regions with equal area.

4. Find the volume of a solid obtained by rotating the region bounded by the given curves about the specified axis.

   a) $x = y - y^2$, $x = 0$ about y-axis;

   b) $y = x$, $y = \sqrt{x}$ about $x = 2$;

   c) $y = \tan^2 x$, $y = 0$, $x = 0$, $x = \pi/4$ about x-axis;

   d) $y = e^x$, $y = e^{-x}$, $x = 1$ about y-axis.

5. Find the volume of the solid S, whose base is a circular disc with radius r and cross-sections perpendicular to the base are squares.

6. A uniform cable hanging over the edge of a tall building is 40 ft long and weights 60 lbs. How much work is required to pull 10 ft of the cable to the top?

7. Find the absolute minimum value of $g(x) = \frac{e^x}{x}$.

8. What is the area of the largest rectangle in the first quadrant with two sides on the axes and one vertex on the curve $y = e^{-x}$?

9. Use the properties of integrals to prove

   a) $\int_0^1 \sqrt{1 + e^{2x}} dx \geq e - 1$;

   b) $\int_0^1 e^x \cos x dx \leq e - 1$.

10. Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

    a) $\int_{-\infty}^{\infty} x^2 e^{-x^3} dx$;

    b) $\int_0^{\infty} \frac{1}{(x+2)(x+3)} dx$;

    c) $\int_0^1 \frac{\ln x}{\sqrt{x}} dx$;

    d) $\int_0^{\pi/4} \frac{\cos x}{\sqrt{\sin x}} dx$.

11. Find the length of the curve

    a) $y = \ln(\sin x)$, $\pi/6 \leq x \leq \pi/3$;

    b) $y^2 = 4x$, $0 \leq y \leq 2$.

12. **a)** Find the Midpoint and Trapezoid approximations M_4, T_4 for $\int_0^1 e^{x^2} dx$.

    b) How large should n be to guarantee that the Midpoint approximation M_n is accurate within 0.001.