MATH 2934 – Additional FFT problem assigned on 2/16/16

Additional problem.

One can use linear approximations to estimate how much the value of a function will change when the values of the arguments of the function change by small amounts. As an example, in this problem you will estimate the amount of metal needed to make a cylindrical can, by using the concept of linearization of a function.

Assume that a closed cylindrical can (like that the ones in which they sell canned beans in supermarkets) has a perfectly cylindrical shape. Let the inside dimensions of the can (i.e., the dimensions of the cavity inside the can) be the following: inside radius $r_0 = 4\,\text{cm}$ and inside height $h_0 = 10\,\text{cm}$. Let the outside dimensions of the can be $r_1 = 4.1\,\text{cm}$ and $h_1 = 10.1\,\text{cm}$. The volume of the metal of which the can is made can be computed as the difference between the "outside" volume of the can, $V_1 = \pi r_1^2 h_1$, and the "inside" volume of the can, $V_0 = \pi r_0^2 h_0$.

(a) Compute the numerical value of the exact volume of the metal needed to make the can, i.e.,

$$(\Delta V)_{\text{exact}} = V_1 - V_0 = \pi r_1^2 h_1 - \pi r_0^2 h_0 \ .$$

(b) Now you will use the linear approximation to the function $V(r,h) = \pi r^2 h$ to approximate the value $(\Delta V)_{\text{exact}}$ found in part (a). Start by finding the partial derivatives $V_r(r,h)$ and $V_h(r,h)$.

(c) Find the numerical values of $V(r_0,h_0)$, $V_r(r_0,h_0)$, and $V_h(r_0,h_0)$, for the values of r_0 and h_0 given above.

(d) Use the linearization $L(r,h)$ of the function $V(r,h)$ at the point $(r_0,h_0) = (4\,\text{cm}, 10\,\text{cm})$, i.e.,

$$L(r,h) = L(r_0,h_0) + V_r(r_0,h_0)(r - r_0) + V_h(r_0,h_0)(h - h_0) \ ,$$

to find the numerical value of $L(r_1,h_1)$ for $(r_1,h_1) = (4.1\,\text{cm}, 10.1\,\text{cm})$.

(e) Find the numerical value of the approximate amount of metal needed to make the can (obtained by using the linear approximation),

$$(\Delta V)_{\text{approx}} = L(r_1,h_1) - L(r_0,h_0) \ .$$

(f) Compute the numerical value of absolute value of the difference

$$|(\Delta V)_{\text{approx}} - (\Delta V)_{\text{exact}}| \ ,$$

and the *relative error* in approximating $(\Delta V)_{\text{exact}}$ by using the linear approximation, i.e., the value of

$$\frac{|(\Delta V)_{\text{approx}} - (\Delta V)_{\text{exact}}|}{(\Delta V)_{\text{exact}}} \ .$$