

In order to get full credit, all answers must be accompanied by appropriate justifications.

Name:

ID#:

1(10)	3(20)	5(30)	7(20)	
2(30)	4(30)	6(30)		total

170 points possible

Problem 1. True/ False statements. Circle the right answer.

(1A) The domain of \tanh is all real numbers.
(True / False)

(1B) The function $f(x) = \cosh(x)$ is injective.
(True / False)

(1C) The function $g(y) = \log_2(y)$ is injective.
(True / False)

(1D) $F(x) = x \ln(x) + x$ is an antiderivative of $f(x) = \ln(x)$.
(True / False)

(1E) $\tan^{-1}(1) = \frac{\pi}{4}$.
(True / False)

(1F) The range of the function \sin^{-1} is all real numbers.
(True / False)

(1G) $\lim_{x \rightarrow \infty} 4^{-x} = \infty$
(True / False)

(1H) $\sinh^2(x) + \cosh^2(x) = 1$ for all x .
(True / False)

(1I) The range of \tanh is $[-1, 1]$.
(True / False)

(1J) The domain of $\log_{1/2}$ is $(0, \infty)$
(True / False)

Problem 2. Find the following limits. Show how you got the answer.
(2A)

$$\lim_{x \rightarrow \infty} x^{1/x}.$$

(2B)

$$\lim_{x \rightarrow \infty} \frac{\ln(\sqrt{x})}{x^2}.$$

(2C)

$$\lim_{x \rightarrow 0^+} \sin(x) \ln(x).$$

Problem 3.

Prove that the function

$$f(x) = \frac{x-1}{x-3}$$

is injective (one-to-one). What is the range of the inverse?

Problem 4

(4A) Find the derivative of

$$y = x^{\cos(x)}$$

(4B) Find y' if

$$y = \ln(x^2 + y^2)$$

(4C) Find the derivative of the function

$$f(s) = \tanh(s) + \frac{3}{s} - \cos^{-1}(s) + \left(\frac{1}{2}\right)^s$$

Problem 5.

(5A) Find the indefinite integral

$$\int \frac{3^{\ln(t)}}{t} dt$$

(5B) Find the definite integral

$$\int_2^5 \frac{1}{1+2r} dr$$

(5C) Find the indefinite integral

$$\int \frac{\sinh(\sqrt{x})}{\sqrt{x}} dx$$

Problem 6.

(6A) Find the indefinite integral

$$\int \frac{\cos(x)}{1 + \sin^2(x)} dx$$

(6B) Find the indefinite integral

$$\int \frac{\sqrt{1 + e^{-x}}}{e^x} dx$$

Problem 7. Prove that for all x and y :

$$\cosh(x + y) = \cosh(x) \cosh(y) + \sinh(x) \sinh(y)$$