Course descriptions, Fall 2023

The following courses are tentatively being offered for the Fall 2023 semester. Descriptions have been given by some instructors; more may be added over time.

MATH 5173 Advanced Numerical Analysis I
Instructor: Ying Wang

Prerequisites: Math 4433, 4443 (Analysis). If you have not taken these courses, but are interested in the course, feel free to get in touch with me. Previous knowledge of Numerical Analysis is **NOT** assumed, but courses like MATH 4073 or MATH 4093/5093, or Electrical Engineering 3793 will be useful complements to this course.

Description: Mathematical models for modern applied sciences and engineerings often consist of ordinary and partial differential equations. In most cases these problems do not have a simple explicit solution, and can only be solved numerically. The construction and the accuracy of the numerical computation become vitally important.

Math 5173 Advanced Numerical Analysis I is an introduction to some of the most important numerical methods for the initial and boundary value problems for the ordinary and partial differential equations. No prior knowledge of numerical methods is assumed. The topics will include (but not limited to) single-step, multi-step, explicit and implicit numerical integrators for nonlinear ODEs, stiff ODEs, Hamiltonian systems (symplectic integrator), Poisson, heat, and wave equations. For example, the finite difference, finite element, and finite volume methods will be discussed. The focus will be on the mathematical aspect of the stability analysis and the order of accuracy study. Knowledge of a programming language is **NOT** a prerequisite for taking the course.

Grading: The grade for the course will be based on regularly assigned homework (about 4 HW sets), and a take-home final exam. The approximate contributions of each type of assignment to the final grade are: homework 60%, final exam 40%.

References:

- *Finite Difference and Spectral Methods for Ordinary and Partial Differential Equations*, by Lloyd N. Trefethen, available online `http://people.maths.ox.ac.uk/trefethen/pdetext.html`.
- *Mathematical analysis*, by Joel Feinstein, available online `https://www.oercommons.org/courses/mathematical-analysis`
- *Numerical Solution of Partial Differential Equations: Finite Difference Methods*, by G. D. Smith, 3rd edition.
- *Numerical Partial Differential Equations: Finite Difference Methods*, by J. W. Thomas, Springer.
- *The Mathematical Theory of Finite Element Methods*, by S. C. Brenner and R. Scott, 3rd edition, Springer, online access through OU library.

MATH 5253 Intro to Mathematics Pedagogy Research
Instructor: Sepideh Stewart

Description: In this course, we will take a practical and hands-on method of preparing you to conduct mathematics education research at the university level. We will study the literature on (a) mathematics education research and (b) qualitative research methods.

Topics:

- Studying a selection of mathematics education research literature
- Constructing mathematics education models, conceptual and theoretical frameworks
- Designing a qualitative study
- Learning about qualitative methods and approaches
- Conducting data collection and analysis
- Writing a qualitative research study

Assessment:

- Literature Review on Mathematics Education Research: 25%
- Writing a research Proposal: 50%
- Presentation: 15%
- Class participation: 10%

MATH 5353 Abstract Algebra I
Instructor: Travis Mandel

MATH 4373/5373 Abstract Linear Algebra
Instructor: Greg Muller

Description: Abstract Linear Algebra rebuilds the tools of linear algebra from an axiomatic foundation. Topics include vector spaces, linear transformations, Jordan canonical form, inner product spaces, and the Spectral Theorem. Students are expected to have taken a linear algebra course which covered the basics of vectors, matrices, bases, and subspaces, but this course will provide students with an opportunity to refresh and deepen their knowledge of these topics.

MATH 5403 Calculus of Variations
Instructor: Nikola Petrov

Description: Calculus of variations is a field of analysis that studies maxima and minima of functionals. Usually functionals of interests are defined as integrals of certain functions, and the goal is to study extremals of such functionals (i.e., functions that minimize or maximize the value of the functional) – their existence, uniqueness, regularity, methods for constructing extremals. Calculus of variations was motivated by physics and has many practical applications.

Topics (tentative):

- Functionals: definition, properties, weak and strong extrema of functionals.
- The first variation: Euler-Lagrange equations, invariance of the Euler-Lagrange equations, Du Bois-Reymond's lemma, particular cases.

- Generalizations: higher-order derivatives, multiple unknown functions, multiple independent variables.
- Constrained systems: Lagrange multipliers, isoperimetric constraints, holonomic constraints.
- The second variation: Legendre and Jacobi conditions, Jacobi fields, conjugate points.
- Variable endpoints: transversality, broken extremals, Weierstrass-Erdmann conditions.
- Canonical formalism: phase space, Hamilton's equations, Poisson brackets, Hamiltonian flow, canonical transformations, Legendre transform.
- Hamilton-Jacobi theory: first-order PDEs, characteristic equations, Hamilton-Jacobi equation, application to geometric optics – Fermat's principle, Huygens construction.
- Conservation laws: local one-parameter transformation groups, generators, variational symmetries, Noether's Theorem.
- Sufficient conditions for a strong extremum: field of extremals, Hilbert invariant integral, Weierstrass $\mathscr{E}$-function.

We will cover many interesting examples from geometry (geodesics, Queen Dido's isoperimetric problem, minimal surfaces), mechanics (particle motion, curve of steepest descent, shape of a hanging cable, motion of strings, membranes, and fluids), and optics (reflection, refraction, caustics).

Please note that no background in physics is expected!

We will review briefly some concepts of calculus as needed (Taylor series, integral theorems of calculus, changes of variables, implicit function theorem, elementary facts about function spaces).

Textbooks: I. M. Gelfand, S. V. Fomin, *Calculus of Variations*, Dover, 1991, $11.79 on Amazon.

We will also use parts of the following books, freely available online for OU students:

- B. van Brunt, *The Calculus of Variations*, Springer, 2004,
- A. Rojo, A. Bloch, *The Principle of Least Action*, Cambridge University Press, 2018,
- H. Kielhöfer, *Calculus of Variations: An Introduction to the One-Dimensional Theory with Examples and Exercises*, Springer, 2018,
- P. Freguglia, M. Giaquinta, *The Early Period of the Calculus of Variations*, Birkhäuser, 2016.

MATH 5423 Complex Analysis I
Instructor: Boris Apanasov

Description: The goal of the course is to make you familiar with methods and technique of complex/ quaternions/ octonions numbers and analysis of functions of complex variables, with emphasis on the geometric approach to the basics. The text is written by the world class mathematician Lars Ahlfors who was the first Fields Medal recipient and is famous for his clear expositions in his books, papers and talks (as I have witnessed in our personal encounters).

No doubts the methods of complex analysis are fundamental for many fields of research, from applied mathematics (including engineering and economics) to pure mathematics and theoretical physics, such as geometry and topology of surfaces and manifolds, algebraic, differential and Riemannian geometries, group representations, differential equations and non-linear analysis, control theory, high energy physics, cosmology, string theory, etc, etc.

In the Fall semester we cover such topics as holomorphic functions, power series, elements of conformal mappings and fractional linear transformations, and complex integration (with calculus of residues).

In the Spring semester we will explore n-dim complex structures. In particular we will study different complex structures in (Siegel) domains in complex spaces and complex manifolds, their biholomorphic mappings (automorphisms), their singularities and related questions. These topics are closely related to different geometries in such Siegel domains and manifolds defined by corresponding Hermitian forms, as well as to algebra of their automorphism groups (especially of the 2-nilpotent Heisenberg groups). This provides a great opportunity to understand similarities and differences between real and complex structures (and their typical objects like $\mathbb{R}$- and $\mathbb{C}$-circles and spheres) as well as CR- and contact structures on manifolds.

This class will combine traditional lectures with an informal setting of research seminars which will involve active students participation and individual projects/presentations. It will be enough space for your own explorations (individual or in groups) of some topics close to your interests. Such research projects with class presentation can be used in lieu of the final exam.

MATH 5453 Real Analysis I
Instructor: Keri Kornelson

MATH 4653/5653 Introduction to Differential Geometry I
Instructor: Nick Miller

MATH 4673/5673 Graph Theory I
Instructor: Max Forester

Description: This is an introductory course on properties and applications of graphs. We will study graph-theoretic concepts such as paths, Eulerian circuits, trees, distance, matchings, connectivity, network flows, colorings, planarity, and spanning cycles.

MATH 4773/5773 Applied Regression Analysis
Instructor: Wayne Stewart

Description: MATH 5773 applies linear algebra to multiple linear regression in order to obtain σ^2 and β estimates. The course starts with categorical variables and moves into hyper-plane column space theory to find interval and point estimates to linear models. While the course is theoretical it is also practical with substantial R development and application to many scientific problems.

MATH 5853 Topology I
Instructor: Jing Tao

MATH 6333 Lie Theory I
Instructor: András Lőrincz

Description: Basic properties of Lie groups and their Lie algebras, structure theorems for complex Lie algebras, nilpotent and solvable Lie algebras, semi-simple Lie algebras, root systems, classification theorems and Dynkin diagrams.

MATH 6393 Topics in Algebra
Instructor: Alan Roche

Description: The course will be an introduction to the representation theory of groups and algebras (almost always finite groups and finite-dimensional algebras) via noncommutative ring theory.

Prerequisites: The qual. sequence in abstract algebra or permission of instructor. The number (6393) is a bureaucratic artifact: it's really a 5000-level course.

Core topics: semisimple modules and rings: Wedderburn-Artin theory; the Jacobson radical; representations of finite groups: characters and orthogonality relations, induction and restriction functors, Mackey theory (how to decompose an induced representation).

I hope also to cover some of the following: representations of symmetric groups and general linear groups; rationality questions; introduction to homological methods.

Sources:

- A first course in noncommutative rings, T.Y. Lam.
- Introduction to representation theory, P. Etingof et al.
- Linear representations of finite groups, J.P. Serre.

MATH 6493 Literacy in Analysis
Instructors: Yan Mary He, Samuel Lin, Jing Tao

Descriptions: (1) He: Anosov representations are generalizations of Fuchsian representations to higher rank Lie groups. We will spend these 5 weeks discussing Anosov representations from dynamical perspectives. In particular, we will discuss metrics on the deformation spaces of Anosov representations, the equivalence of Anosov representation and dominated splitting and the Patterson-Sullivan theory of Anosov representations.

(2) Lin: The hyperbolic space and its geometry are of interest to mathematicians in many different areas, including differential geometry, topology, and number theory. One of the most fundamental results in hyperbolic geometry is the MostowâĂŹs rigidity theorem, which states that two compact hyperbolic manifolds of dimension greater than two are isometric if and only if their fundamental groups are isomorphic. In these five weeks, we will give a brief introduction to the geometry of hyperbolic spaces, and then present two proofs for the theorem, one based on Gromov-Thurston and the other based on Besson-Courtois-Gallot.

(3) Tao: This is an introduction to Measurable Group Theory, which is the study of infinite countable groups using measure-theoretical tools. The primary examples are lattices in Lie groups, such as SL(n,Z). In these 5 week, we will take a historical tour through the seminal results in the study of lattices that led to the development of this burgeoning field.

MATH 6673 Differential Geometry I
Instructor: Michael Jablonski

Description: Among the most basic and wide spread concepts in modern mathematics is that of a manifold. The first semester of this sequence will be a course dedicated to manifold theory. The only requisites are the basic first year courses that all graduate students take.

Topics:

0. Review of differential calculus
    - Inverse function theorem and constant rank theorem
    - Applications

1. Manifolds

    - Coordinate charts and atlases, differential structures
    - Tangent vectors and tangent spaces
    - Differential maps
    - Submanifolds, immersions, submersions
    - Covering maps
    - Orientable and nonorientable manifolds

2. Vector bundles

    - Tangent and cotangent bundles
    - Tensor bundles and general bundle constructions

3. Vector fields

    - Vector fields as sections of the tangent bundle, as derivatives of functions
    - Existence and uniqueness of integral curves
    - Flows
    - Lie derivative of vector fields and 1-forms
    - Lie bracket and Frobenius theorem
    - Extensions of vector fields defined on manifolds

4. Differential forms

    - Forms as sections of the bundle of kth exterior powers of T*M
    - Wedge products, interior products, exterior derivative, pullbacks
    - Lie derivative of k-forms
    - Integration of forms on manifolds, Stokes Theorem
    - Change of variables formula (differential forms version)
    - Frobenius Theorem (differential forms version)

5. Lie groups

    - Definition and examples
    - Subgroups and subalgebras
    - Left and right invariant forms, volume elements
    - Compact Lie groups: biinvariant forms and Haar measure
    - Principal bundles

6. De Rham cohomology

- Definition
- Poincare Lemma
- Induced maps on cohomology and homotopy invariance
- Mayer-Vietoris sequence
- Biinvariant representatives for cohomology classes of compact Lie groups
- Computation of some cohomology groups: spheres, compact surfaces, compact Lie groups

7. Transversality theory

    - Transversality of submanifolds and their mappings
    - Regular values, Sard's theorem, degree of a map
    - Approximation of maps by immersions and imbeddings