

Syllabus for Real Analysis I and II (Math 5453-63)

Masters and Ph.D. Qualifying Examination

L. W. White, 2004

1. Metric spaces: Elements of the theory of metric spaces. Complete metric spaces, Baire category theorem and applications. Structure of open and closed subsets of the reals.
2. Lebesgue Measure on \mathbb{R} : Outer measure, Caratheodory's characterization of Lebesgue measurable sets, approximations by open and closed sets, F_σ and G_δ . Existence of nonmeasurable sets.
3. Abstract Measure spaces: σ -algebras of sets, measures defined on σ -algebras. Elementary properties of measures.
4. Measurable Functions: Definition and basic properties of measurable functions, approximation by simple measurable functions. Sequences of measurable functions, modes of convergence (pointwise, pointwise a.e., almost uniform, convergence in measure). Egoroff's theorem, Lusin's theorem, and the Weierstrass approximation theorem.
5. The Lebesgue Intergral: Definition and basic properties, Monotone convergence theorem, Fatou's lemma, dominated convergence theorem. Equality of the Riemann and Lebesgue integrals for Riemann integrable functions.
6. Differentiation: Vitali covering theorem, Dini derivates, monotone functions, functions of bounded variation, absolutely continuous functions. Characterization absolutely continuous functions as indefinite Lebesgue integrals of their derivatives.
7. L^p spaces: Holder and Minkowski inequalities, L^p convergence and its relations to other types of convergence. Completeness of L^p . Dense subsets of L^p . The dual space of norm linear space and the Riesz representation theorem for L^p .
8. Signed measures: Hahn and Jordan decomposition theorems, Radon-Nicodym theorem, Lebesgue decomposition theorem.
9. Product measures: Definition of product measures, Tonelli and Fubini theorems.
10. Convolutions: Approximations of identity.

References: H. L. Royden, *Real Analysis* (3rd edition)

R. Bartle, *The Elements of Integration*

W. Rudin, *Real and Complex Analysis* (2nd edition)

Boas, *Primer on Real Functions*