Qualifying Exam
Real Analysis

August, 2007

Name: ____________________ **ID#** ____________________

ATTENTION! Do any 6 of the following 7 problems. Circle the problems you submit for grading.

1.
a). Define: Set E is Lebesgue measurable in $\mathbb{R}$.
b). Prove: If E_1 and E_2 are Lebesgue measurable, then so is $E_1 \cup E_2$.
c). Prove: Lebesgue measurable sets form an algebra.

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2. State and prove Riesz-Fischer Theorem.

3.
a). State and prove Kakutani-Krein Theorem.
b). State and prove Stone-Weierstrass theorem.

4.
a). Define: μ is a signed measure on X.
b). State Hahn Decomposition Theorem.
c). State Jordan Decomposition Theorem.
d). State and prove Lebesgue Decomposition Theorem.

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5. State and prove Riesz Representation Theorem for the dual of $L^p(X)$, $1 \leq p < \infty$.

6.
a). Define: F is a positive linear functional on $C(X)$.
b). Prove: If X is a compact metric space and $F \in [C(X)]^*$, then there exist positive linear functionals F^+ and F^- on $C(X)$ such that

$$F = F^+ - F^-$$

and $\|F\| = F^+(1) + F^-(1)$.

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7. Let X be a compact metric space. State and prove Riesz-Markov Theorem for the dual of $C(X)$.