Analysis Qualifying Exam Syllabus

03/21/2011

The real analysis qualifying exam will be based on material related to the topics listed below. This list is not meant to be exhaustive, but is intended to be a guide to the subjects to be studied thoroughly.

Metric Spaces and Topology

- metric spaces, open and closed sets, convergence of sequences, completeness, compactness
- continuous functions, uniform continuity, uniform convergence of functions.
- Baire category theorem and applications, Arzela-Ascoli theorem, Stone-Weierstrass theorem

Measure and Integration

- measure and outer measure on abstract measure spaces
- Lebesgue outer measure, Lebesgue measurable sets, Lebesgue measure in $\mathbb{R}$ and $\mathbb{R}^n$
- Lebesgue-Stieltjes measure dg for a function g of bounded variation
- signed measures, Hahn and Jordan decomposition theorems
- measurable functions, integral of a measurable function with respect to a measure
- Monotone Convergence Theorem, Fatou's lemma, Lebesgue Dominated Convergence theorem
- Lebesgue integral, Lebesgue-Stieltjes integral
- dense subspaces of L^1, e.g. simple functions, continuous functions with compact support
- characterization of Riemann integrable functions, equality of Lebesgue and Riemann integrals
- Radon-Nikodym theorem, Lebesgue decomposition theorem
- convergence in measure, Egorov's theorem, Lusin's theorem
- Fubini's theorem, Tonelli's theorem

Differentiation

- Dini derivates, derivatives of monotone functions, derivatives of indefinite integrals
- functions of bounded variation, absolutely continuous functions, the Cantor-Lebesgue function, characterization of absolutely continuous functions as the integrals of their derivatives

L^p spaces

- Hölder and Minkowski inequalities
- Banach spaces, dense subspaces of L^p, separability of L^p, completeness of L^p
- L^p duality/Riesz Representation Theorem (representation of linear functionals on L^p)
- Hilbert spaces, L^2 spaces.

References

- C. Apostol, Mathematical Analysis
- G. Folland, Real Analysis
- H. Royden (or Royden, Fitzpatrick), Real Analysis
- W. Rudin, Real and Complex Analysis