Syllabus for Topology Qualifying Exam, 2016

The 2015-2016 topology graduate course used the books Topology (second edition) by Munkres and Algebraic Topology by Hatcher (chapters 0 and 1).

General Topology

I. Topological spaces and continuous maps (Munkres, sections 12-22)

1. Topological spaces, bases, subbases
2. The order topology
3. The product topology (two factors)
4. The subspace topology
5. Closed sets and limit points, Hausdorff spaces
6. Continuous maps, homeomorphisms, local continuity, pasting lemma, maps into products
7. The product topology (general case), maps into product spaces, box topology
8. Metric spaces, uniform topology
9. The quotient topology, maps out of quotient spaces

II. Connectedness and compactness (Munkres, sections 23-29)

1. Connected spaces, connectedness of products
2. Connectedness in linear continua, intermediate value theorem, path connectedness
3. Components and local connectedness
4. Compact spaces: continuous maps, products, tube lemma, finite intersection property
5. Extreme value theorem, Lebesgue number lemma
6. local compactness, limit-point compactness, one-point compactification
7. Compactness of $I = [0, 1]$ is compact (using only the least-upper bound property of $\mathbb{R}$)

III. Countability and separation axioms (Munkres, sections 30-32)

1. First and second countability axioms
2. regular spaces, normal spaces

IV. Urysohns Lemma and applications (Munkres, sections 33-36)

1. Urysohns Lemma, separation by continuous functions (Construction of Urysohn function)
2. Tietze extension theorem
3. Partitions of unity
4. Embeddings of manifolds

V. Tychonoff theorem (statement and applications only) (Munkres, sections 37)

VI. Complete metric spaces and Function spaces (Munkres, sections 43, 44, 46)

1. uniform metric
2. space-filling curve
3. topology of pointwise convergence (= point-open topology)
4. The compact-open topology, the evaluation map, induced maps

Algebraic Topology

I. The fundamental group (Munkres, sections 51-60; Hatcher, section 1.1)

   1. Path homotopy, properties
   2. Fundamental group, induced homomorphisms
   3. Fundamental group of the circle (via a covering space)
   4. Retractions and the fundamental group, Brouwer fixed point theorem
   5. The Borsuk-Ulam theorem, applications
   6. Deformation retractions, homotopy equivalences

II. Covering spaces (Hatcher, section 1.3; Munkres, sections 53-54, 79-82)

   1. Definition of covering spaces
   2. Path lifting and uniqueness
   3. Injectivity of induced map on fundamental group
   4. The lifting criterion, uniqueness of lifts
   5. Construction of covering space (For a subgroup H of $\pi_1(B, b_0)$, a covering space $p\colon (E, e_0) \to (B, b_0)$ such that $p_*(\pi_1(E, e_0)) = H$; Describe the space E, its topology and the map p, and an evenly covered neighborhood of any $b \in B$).
   6. Equivalence of covering spaces, correspondence between subgroups and covering spaces
   7. Covering transformations, regular covering spaces

III. The van Kampen theorem (Hatcher, section 1.2 and chap. 0; Munkres, sections 69-73)

   1. Free products of groups, existence, the mapping property
   2. The van Kampen theorem
   3. 1- and 2-dimensional cell complexes, attaching cells, collapsing a contractible subcomplex
   4. Generators and relations, fundamental groups of cell complexes

IV. . Group actions (Hatcher, section 1.3)

   1. Properly discontinuous action
   2. Orbit spaces

V. Graphs and free groups (Hatcher, section 1.A)

   1. Graphs = 1-dimensional cell complexes
   2. Cayley graph, Cayley complex of a group, presentation complex of a group