ALGEBRA QUALIFYING EXAM SYLLABUS 2020

The course text was *Abstract Algebra* (second edition) by Pierre Grillet (Springer 2007). We covered most of Chaps. I - V + VIII. Other recommended sources:

- *Algebra: a graduate course* by I. Martin Isaacs (Amer. Math. Soc. 2009);
- *Abstract Algebra* (third edition) by David Dummit and Richard Foote (Wiley 2004).

If you have questions, you're welcome to write to `aroche@ou.edu`.

Groups

- Groups, subgroups, cosets, Lagrange's Theorem;
- Homomorphisms, normal subgroups, quotient groups, isomorphism theorems;
- Subgroup structure of cyclic groups;
- Group actions, orbits and stabilizers, class equation;
- Symmetric and alternating groups;
- Sylow Theorems;
- Composition series and the Jordan-Hölder Theorem;
- Solvable and nilpotent groups;
- Semidirect products, group extensions;
- Free groups and group presentations.

Rings

- Rings, subrings, ideals;
- Homomorphisms and quotient rings;
- Domains and fields, prime ideals and maximal ideals;
- Polynomial rings;
- Principal ideal domains and unique factorization domains;
- Factorization of polynomials, Gauss' Lemma, Eisenstein criterion;
- Noetherian rings, Hilbert Basis Theorem;
- Comaximal ideals and the Chinese Remainder Theorem.

Fields and Galois Theory

- Field extensions – algebraic and transcendental elements, degree, minimal polynomials;
- Splitting fields, separable and inseparable extensions;
- Classification of finite fields;
- Normal and Galois extensions;

- Primitive element theorem;
- Galois correspondence – existence, properties, examples;
- Cyclotomic fields;
- Solvability by radicals;
- Some Galois group computations.

Modules

- Definition and examples;
- Homomorphisms of modules, isomorphism theorems;
- Simple modules, annihilators;
- Direct sums, free modules;
- Structure of finitely generated modules over PIDs;
- Rational and Jordan canonical form.