Syllabus for Qualifying Exam, 2020

The 2019-2020 topology graduate course and qualifying exam used as reference the books *Topology* (Second edition) by Munkres and chapter 1 of *Algebraic Topology* by Hatcher.

1. Point Set Topology

1. Metric spaces: metric topology, sequences, limits, compactness in metric spaces (including the different equivalent definitions/characterizations), completeness, totally bounded
2. Basic definitions/operations: subspace topology, basis, subbasis, continuous functions, homeomorphisms, pasting lemma, closed sets and limit points, closed map, open map, interior, closure, boundary, separable
3. Products: box and product topology on finite and infinite products, Tychonoff's Theorem (statement and applications), continuous maps into products
4. Separation Axioms: Hausdorff, normal, regular, Urysohn Lemma, Urysohn Metrization Theorem
5. Connectedness, Path-connectedness, connectedness of products, connected components, path components, totally disconnected spaces
6. Compactness: various kinds of compactness (compact, sequentially compact, limit point compact), images under continuous maps, Lebesgue number, tube lemma, compact sets in $\mathbb{R}^n$; relation between compactness, Hausdorff property, and closed subsets of compact spaces
7. Quotient topology, quotient spaces, maps from quotient spaces, quotient maps
8. 2nd countable, Baire space, Baire category theorem
9. local compactness and compactifications
10. Space-filling curve

2. Fundamental Group and Covering Spaces

1. Hatcher, Chapter 0. Homotopy and homotopy equivalence, CW complexes, Retractions and deformation retractions

2. Hatcher, Chapter 1.1. Paths and homotopies of paths, definition and examples of fundamental group, induced homomorphisms, change of basepoint isomorphisms for fundamental group

3. Hatcher, Chapter 1.2. van Kampen's theorem and examples, fundamental group of CW complex, fundamental group of wedge sum of spaces

4. Hatcher, Chapter 1.3. Definition and basic properties of covering spaces (e.g., lifting paths, homotopies, and continuous maps), classification of covering spaces (Galois correspondence), universal cover, regular/normal covers including examples, irregular covers including examples, group actions and deck transformations

5. Other concepts/definitions: semilocally simply connected, locally path connected, simply connected, contracible

6. Brouwer fixed point theorem in dimension 2