The course text was *Basic Algebra I* (second edition) by Nathan Jacobson. We covered most of Chapters 1–4 of this text, as well as parts of Chapters 6–7, together with a small amount of supplementary material.

Here is a list of sections of the text that may be covered on the qualifying exam. You may contact Kimball Martin if you have more specific questions.

1 MONOIDS AND GROUPS
1.1 Monoids of transformations and abstract monoids
1.2 Groups of transformations and abstract groups
1.3 Isomorphism. Cayley's theorem
1.4 Generalized associativity. Commutativity
1.5 Submonoids and subgroups generated by a subset. Cyclic groups
1.6 Cycle decomposition of permutations
1.7 Orbits. Cosets of a subgroup
1.8 Congruences. Quotient monoids and groups
1.9 Homomorphisms
1.10 Subgroups of a homomorphic image. Two basic isomorphism theorems
1.11 Free objects. Generators and relations
1.12 Groups acting on sets
1.13 Sylow's theorems

2 RINGS
2.1 Definition and elementary properties
2.2 Types of rings
2.3 Matrix rings
2.4 Quaternions
2.5 Ideals, quotient rings
2.6 Ideals and quotient rings for Z
2.7 Homomorphisms of rings. Basic theorems
2.8 Anti-isomorphisms
2.9 Field of fractions of a commutative domain
2.10 Polynomial rings
2.11 Some properties of polynomial rings and applications
2.12 Polynomial functions
2.13 Symmetric polynomials
2.14 Factorial monoids and rings
2.15 Principal ideal domains and Euclidean domains
2.16 Polynomial extensions of factorial domains

3 MODULES OVER A PRINCIPAL IDEAL DOMAIN
3.1 Ring of endomorphisms of an abelian group
3.2 Left and right modules
3.3 Fundamental concepts and results
3.4 Free modules and matrices
3.5 Direct sums of modules

3.6 Finitely generated modules over a p.i.d. Preliminary results
3.7 Equivalence of matrices with entries in a p.i.d.
3.8 Structure theorem for finitely generated modules over a p.i.d.
3.9 Torsion modules, primary components, invariance theorem
3.10 Applications to abelian groups and to linear transformations
3.11 The ring of endomorphisms of a finitely generated module over a p.i.d.

4 GALOIS THEORY OF EQUATIONS
4.1 Preliminary results, some old, some new
4.2 Construction with straight-edge and compass
4.3 Splitting field of a polynomial
4.4 Multiple roots
4.5 The Galois group. The fundamental Galois pairing
4.6 Some results on finite groups
4.7 Galois' criterion for solvability by radicals
4.8 The Galois group as permutation group of the roots
4.9 The general equation of the nth degree
4.10 Equations with rational coefficients and symmetric group as Galois group
4.11 Constructible regular n-gons
4.12 Transcendence of e and pi. The Lindemann-Weierstrass theorem
4.13 Finite fields
4.14 Special bases for finite dimensional extensions fields
4.15 Traces and norms
4.16 Mod p reduction

6 METRIC VECTOR SPACES AND THE CLASSICAL GROUPS
6.1 Linear functions and bilinear forms
6.2 Alternate forms
6.3 Quadratic forms and symmetric bilinear forms

7 ALGEBRAS OVER A FIELD
7.1 Definition and examples of associative algebras
7.3 Regular matrix representations of associative algebras. Norms and traces
7.7 Frobenius' and Wedderburn's theorems on associative division algebras