ALGEBRA QUALIFYING EXAM SYLLABUS FOR 2022

The following list of topics is meant to be representative but not necessarily exhaustive. All of these topics are covered in the textbook Abstract Algebra by David S. Dummit and Richard M. Foote, 3rd edition. It is possible that some of these topics were not covered (in detail) during the course - you should review the list below carefully.

Groups
- Groups, subgroups, homomorphisms
- Cosets, Lagranges Theorem
- Normal subgroups, kernels, quotients, isomorphism theorems
- Group actions, orbits, stabilizers, centralizers, normalizers, class equation
- Cyclic groups, dihedral groups, symmetric groups, alternating groups, matrix groups
- Free groups, presenting groups by generators and relations
- Direct products, semi-direct products, automorphism groups, composition series
- Sylow Theorems, groups of small order
- Classification of finite abelian groups

Rings
- Rings, subrings, homomorphisms
- Ideals, kernels, quotient rings, isomorphism theorems
- Prime ideals, maximal ideals
- Integral domains, rings of fractions
- Chinese Remainder Theorem, Euler ϕ-function, $(\mathbb{Z}/n\mathbb{Z})^{\times}$
- Euclidean domains
- Polynomial rings, polynomial division algorithm
- Principal ideal domains (PIDs), Noetherian rings, Hilbert Basis Theorem
- Irreducible elements, Unique Factorization Domains (UFDs), Gauss Lemma

Modules
- Modules, homomorphisms of modules, categories of modules
- Modules of a polynomial ring, relation with matrices
- Direct sums of modules, extensions of modules
- Tensor products
- Classification of modules over PIDs. Rational and Jordan canonical forms

Fields
- Fields, homomorphisms of fields, standard examples, characteristic of a field
- Field extensions, degree of an extension
- Simple extensions, algebraic extensions, splitting fields, algebraic closures
- Automorphism groups and Galois groups of field extensions
- Separable extensions, Galois extensions, characterizations of Galois extensions
- Galois correspondence, i.e. the Fundamental Theorem of Galois Theory
- Applications to solvability of polynomials and ruler-and-compass constructions
- Extensions of finite fields, the Frobenius endomorphism
- Cyclotomic polynomials and extensions