Syllabus for Topology Qualifying Exams, Fall 2022 and Spring 2023

The 2021-2022 topology graduate course and qualifying exam used as reference the books *Topology* (Second edition) by Munkres and chapter 1 of *Algebraic Topology* by Hatcher.

1 Point Set Topology

1. Metric spaces: metric topology, sequences, limits, compactness in metric spaces (including the different equivalent definitions/characterizations), completeness, totally bounded.

2. Basic definitions/operations: subspace topology, basis, subbasis, continuous functions, homeomorphisms, pasting lemma, closed sets and limit points, closed map, open map, interior, closure.

3. Products: box and product topology on finite and infinite products, Tychonoff's Theorem (statement and applications), continuous maps into products.

4. Connectedness: Path-connectedness, connectedness of products, connected components, path components, totally disconnected spaces, local connectedness.

5. Compactness: various kinds of compactness (compact, sequentially compact, limit point compact), images under continuous maps, Lebesgue number, tube lemma, compact sets in $\mathbb{R}^n$; relation between compactness, Hausdorff property, and closed subsets of compact spaces, local compactness and one point compactifications.

6. Quotient topology: quotient spaces, quotient maps, maps from quotient spaces.

7. Countability and Separation Axioms: First and Second Countable, separable, Lindeloff, Hausdorff, normal, regular.

8. Metrization/Embedding Theorems: Urysohn Lemma, Urysohn Metrization Theorem, topological manifolds, partitions of unity for compact manifolds, embedding compact manifolds into euclidean space.

9. Topologies on function spaces: pointwise convergence, uniform convergence on compact subspaces, compact-open topology, applications to space-filling curves, completions of metric spaces.

2 Fundamental Group and Covering Spaces

1. Hatcher, Chapter 0. Homotopy and homotopy equivalence, CW complexes, Retractions and deformation retractions

2. Hatcher, Chapter 1.1. Paths and homotopies of paths, definition and examples of fundamental group, induced homomorphisms, change of basepoint isomorphisms for fundamental group

3. Hatcher, Chapter 1.2. van Kampen's theorem and examples, fundamental group of a CW complex, fundamental group of a wedge sum of spaces.

4. Hatcher, Chapter 1.3. Definition and basic properties of covering spaces (e.g., lifting paths, homotopies, and continuous maps), classification of covering spaces (Galois correspondence), classification of covering spaces (via right G-sets), universal cover, regular/normal covers including examples, irregular covers including examples, group actions and deck transformations.

5. Other concepts/definitions: semilocally simply connected, locally path connected, simply connected, contractible

6. Brouwer fixed point theorem in dimension 2, Borsuk-Ulam Theorem in dimension 2.