

ALGEBRA QUALIFYING EXAM SYLLABUS

Below is a list of topics that may be covered on the qualifying exam:

Groups

- Groups, cosets, quotients, homomorphisms, isomorphism theorems
- Group actions, orbits, stabilizers, normalizers, centralizers, class equation
- Finitely generated abelian groups, cyclic groups, permutation groups, matrix groups
- Solvable groups, nilpotent groups, simple groups, free groups, group presentations, semi-direct products
- Lagrange's theorem, p -groups, Sylow theorems

Rings and Modules

- Rings, ideals, quotients, homomorphisms, isomorphism theorems
- Maximal ideals, prime ideals, principal ideals, field of fractions
- Integral domains, Euclidean domains, PIDs, UFDs, Noetherian rings, polynomial rings
- Prime and irreducible elements, Gauss's lemma, Eisenstein's criterion
- Modules, quotient modules, tensor product of modules, isomorphism theorems
- Structure of modules over PIDs, rational and Jordan canonical forms, fundamental theorem of finitely generated abelian groups

Fields

- Fields, characteristic, field extensions, degree, algebraic closure
- Algebraic and transcendental elements, minimal polynomial
- Splitting fields, separable polynomials, primitive element theorem
- Cyclotomic fields, finite fields, Frobenius automorphism, function fields
- Norm and trace
- Normal extension, Galois extension
- Galois group, Galois correspondence, solvability by radicals

Here is a non-exhaustive list of textbooks that cover these topics:

- *Abstract Algebra* by David S. Dummit and Richard M. Foote, 3rd edition.
- *Algebra: A Graduate Course* by I. Martin Isaacs.
- *Basic Algebra I* by Nathan Jacobson, 2nd edition.
- *Abstract Algebra* by Pierre Grillet, 2nd edition