ANALYSIS QUALIFIER EXAM SYLLABUS TOPICS

1 Metric spaces and continuous functions

- Metric spaces, completeness, compactness.
- Arzelà-Ascoli theorem (metric space version).

2 Measure and integration

- Algebras, σ-algebras.
- Measures and outer measures on abstract spaces, Borel measures, properties of measures, Carathéodory extension theorem.
- Lebesgue measure on on $\mathbb{R}$, examples of nonmeasurable sets, Lebesgue-Stieltjes measures on $\mathbb{R}$.
- Measurable functions, simple functions, simple function approximation.
- Integral with respect to a measure, monotone convergence theorem, Fatou's lemma, dominated convergence theorem.
- Almost everywhere convergence, convergence in measure, convergence in L_1, relations between different modes of convergence, Egorov's theorem, Lusin's theorem.
- Characterization of Riemann integrable functions, equality of Lebesgue and Riemann integrals.
- Measurability on cartesian product, product measures, Lebesgue measure on $\mathbb{R}^n$, Fubini-Tonelli theorems.
- Signed measures, Hahn and Jordan decomposition theorems, absolute continuity between measures, Radon-Nikodym theorem, Lebesgue decomposition theorem.
- Functions of bounded total variation, absolutely continuous functions, the fundamental theorem of calculus for Lebesgue integrals, integration by parts formula for Lebesgue integrals, C^1 change of variable formula for Lebesgue integrals, Lebesgue differentiation theorem.

3 L_p spaces

- L_p space, duality, completeness, approximation by continuous functions.
- Jensen's inequality, Markov's/Chebyshev's inequality, Hölder's inequality, Minkowski's inequality.

4 Basic Hilbert space and Banach space theory

- Hilbert spaces, Cauchy-Schwarz inequality, projection theorem, Riesz representation theorem for Hilbert spaces, orthonormal basis, Bessel's inequality.
- Banach spaces, dual space.

Bibliography

- Apostol, *Mathematical Analysis.*
- Axler, *Measure, Integration & Real Analysis.*
- Bass, *Real Analysis for Graduate Students.*
- Dudley, *Real Analysis and Probability.*
- Folland, *Real Analysis.*
- Lieb and Loss, *Analysis.*
- Royden, *Real Analysis.*
- Royden and Fitzpatrick, *Real Analysis.*
- Rudin, *Real and Complex Analysis*
- Stein and Shakarchi, *Real Analysis.*
- Tao, *An Introduction to Measure Theory.*
- Wheeden and Zygmund, *Measure and Integral: An Introduction to Real Analysis.*