Topics for Qualifying Exam in Topology (Last update: 10-23-2023)

This is the syllabus for the qualifying exam in topology. Some possible textbooks are *Topology* (second edition) by Munkres and *Algebraic Topology* by Hatcher (chapters 0 and 1). The book *Algebraic Topology: An Introduction* by Massey is also recommended, as it provides more detail than Hatcher in some areas. Some additional resources are listed in the second page.

General Topology

I. Topological spaces and continuous maps (Munkres, sections 12-22)

- Topological spaces, bases, subbases, order topology, subspace topology
- Closed sets and limit points, Hausdorff spaces
- The product topology, maps into product spaces, box topology
- Continuous maps, homeomorphisms, local continuity, pasting lemma, maps into products
- Metric spaces, uniform topology
- The quotient topology, quotient spaces, quotient maps, maps out of quotient spaces

II. Connectedness and compactness (Munkres, sections 23-29)

- Connected spaces, connectedness of products, components and local connectedness
- Connected subspaces in $\mathbb{R}$, intermediate value theorem, path connectedness
- Compact spaces: continuous maps, products, tube lemma, finite intersection property
- Various notions of compactness (compact, sequentially compact, limit point)
- Extreme value theorem, Lebesgue number lemma
- Local compactness, one-point compactification

III. Countability and separation axioms (Munkres, sections 30-32)

1. First and second countability axioms
2. Separable, Hausdorff, regular spaces, normal spaces

IV. Urysohns Lemma and applications (Munkres, sections 33-36)

- Urysohn Lemma
- Urysohn Metrization Theorem (statement only)

V. Other topics (Munkres, sections 37, 46)

- Tychonoff theorem (statement only)
- Topologies on function spaces: pointwise convergence, uniform convergence on compact subsubspaces, compact-open topology, the evaluation map, induced maps

Algebraic Topology

I. Some basic geometric notions (Hatcher, Chapter 0)

- Homotopy and homotopy equivalences,
- CW complexes
- Retractions and deformation retractions

II. The fundamental group (Hatcher, section 1.1; Munkres, sections 51-60;)

- Paths and homotopies of paths, properties
- Fundamental group, induced homomorphisms
- Fundamental group of the circle (via a covering space)
- Brouwer fixed point theorem, Borsuk-Ulam theorem, applications

III. Van Kampen theorem (Hatcher, section 1.2; Munkres, sections 69-73)

- Free products of groups
- Van Kampen theorem and examples
- Fundamental groups of CW complexes

IV. Covering spaces (Hatcher, section 1.3; Munkres, sections 53-54, 79-82; Massey chapter 5, sections 3-6)

- Definition and basic properties covering spaces
- Path lifting and uniqueness
- Injectivity of induced map on fundamental group
- The lifting criterion, uniqueness of lifts
- Classification of coverings spaces (Galois correspondence)
- Universal cover, semilocally simply connected, locally path connected, simply connected, contractible
- Regular and irregular covering spaces including examples
- Group actions and deck transformations

Books

- *Topology* (2nd edition), J. Munkres
- *Algebraic Topology*, A. Hatcher
- *Algebraic Topology: An Introduction*, W. Massey
- *Counterexamples in Topology*, Steen and Seebach
- *Topology*, K. Jänich and S. Levy
- *A First Course in Algebraic Topology*, C. Kosniowski