Math Circle: Exploring the World of Primes

Welcome to the World of Primes!

Hello everyone! Today, we are diving into one of the most fascinating areas of mathematics: **prime numbers**. Primes are the building blocks of all numbers — every whole number greater than 1 is either a prime or can be broken down into a product of primes. Think of them as the "atoms" of arithmetic!

What We Will Do Today

In this session, we will:

- Discover **what prime numbers are** and why they are special.
- Explore **patterns and puzzles** involving primes.
- Engage in **hands-on activities and challenges** that spark creativity.
- Learn surprising facts, like how primes show up in nature.

Whether you are new to primes or have already played with them, today's math circle is about curiosity, exploration, and teamwork. Get ready to test your problem-solving skills, share ideas, and see the magic of prime numbers in action!

Prime numbers and Composite numbers

1. We can write $4 = 4 \times 1 = 2 \times 2$, are there any more ways to write 4?
2. We say 2 is a divisor of 4 because we can write 4 as 2 times something.

Write down all the divisors of the following numbers

1. 16

2. 21

3. 11

Given any number m what two numbers are always divisors of m?

We call a number a **prime number** if it has only 2 divisors. Can you write down a few prime numbers from above?

A number is called a **composite number** if it has more than 2 divisors. Can you write a few composite numbers from above?

Is 1 prime or composite?

Let us try to determine if the following numbers are prime

1. 53

2. 55

3. 56

Patterns in prime numbers

Hopefully now you are convinced that prime numbers are important. Let us explore more properties of prime numbers

1. How many prime numbers are there?

2. Notice that $2^2 - 1 = 3$ and $2^3 - 1 = 7$ are primes. Is it always true that $2^n - 1$ is a prime?

3. The largest prime numbers currently known to us $2^{136,279,841} - 1$.
   This number has 41,024,320 digits. Using standard A4 paper format of 50 lines per page and 75 characters per line, it would require 10,940 single-sided pages to print this prime number, or approximately 22 reams of paper.
   The .txt file housing this number takes up 41.8Mb. For reference, .txt file for Leo Tolstoy's War and Peace (587,287 words) is a paltry 3.4 Mb.

4. Calculate $n^2 + n + 41$ for $n = 1, 2, 3, 4, 5$. What if $n = 41$?

In general finding prime numbers is fairly difficult. Let us build a machine which will give us all the prime numbers from 1 to 100.

Hunting for primes

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1. Start with the number 2, circle it since 2 is prime. Cross out all the multiples of 2.
2. Move on to the next uncrossed number, 3. This is a prime number, so circle it. Cross out all multiples of 3.
3. Repeat this process until all boxes are crosses/circled.
4. All the circled numbers are primes and the crossed out ones are composite.

Prime numbers in nature

Every 13 or 17 years, huge swarms of periodical cicadas appear in certain parts of North America. These insects stay underground as nymphs for most of their lives, then emerge in massive numbers to mate and lay eggs. The predators of cicadas have shorter life cycles like $2-5$ years. Scientists have noticed that cicadas choose 13 or 17 year life cycles. Why might this be the case?

1. Imagine a predator appears every 2 years and the cicadas appear every 12 years? How often do they meet?

2. What if predator appears every 3 years and the cicadas appear every 5 years?

Prime-numbered life cycles help cicadas avoid predators, showing how primes provide clever strategies even in nature.

Divisibility tests

Let us explore some common divisibility rules.

(a) How do you check if a number is divisible by 2? What about 4?

(b) How do you check if a number is divisible by 3?

(c) How do you check if a number is divisible by 5?

(d) How do you check if a number is divisible by 7?

(e) How do you check if a number is divisible by 9?

(f) How do you check if a number is divisible by 11?

Some warmup problems

1. What is the largest six-digit number that is NOT divisible by 2, 3, 5, or 11?

2. Find the largest four-digit palindrome that is divisible by 3.

3. Find the largest five-digit palindrome that is divisible by 6.