

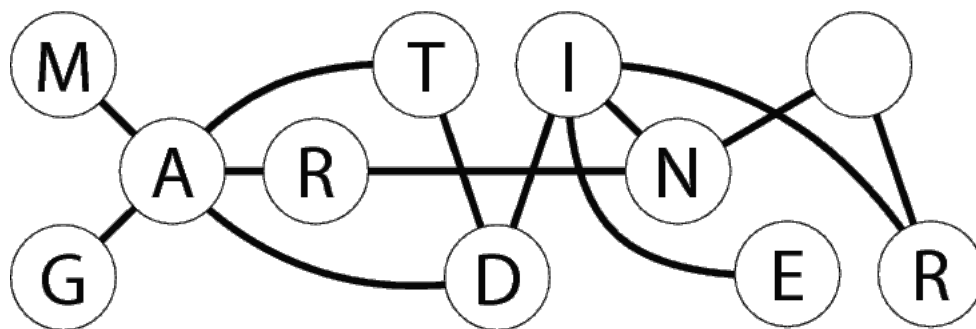


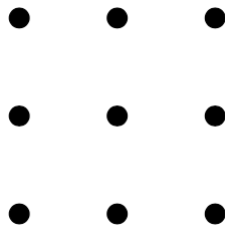
Photo Martin Gardner by Alex Bellos in 2008 in Norman

Born in Tulsa in 1914 and passed away in Norman in 2010.

Stage 1

Stage 1, Round 1 (2 Questions, 3 Minutes)

1. If you are not allowed to pick three dots in the same row, same column, or same diagonal, what is the maximum number of dots you can pick in a three by three grid of dots?



2. To convert from Fahrenheit (F) to Celsius (C), you use the formula

$$C = \frac{5}{9}(F - 32).$$

There is a temperature which is exactly the same number in both Fahrenheit and Celsius. What is that temperature?

Stage 1, Round 2 (Blitz Round, 3 Minutes)

- a. If we write the number A in binary (base 2) it is 11. What is A ?
- b. If $\sec(\theta) = 5/3$, then what is $\sin(\theta)$?
- c. If you have a spherical birthday cake and make three distinct straight cuts through the cake, what is the largest number of pieces you can make?
- d. Let $x = 11^{10}$. Which of the following numbers is closest to x ?
 - (a) 1,000,000
 - (b) 1,000,000,000
 - (c) 1,000,000,000,000
 - (d) 1,000,000,000,000,000
- e. Which is more likely: rolling two sixes with two rolls of a fair die or drawing the four of spades from a standard deck of 52 playing cards which has been well-shuffled?

Stage 1, Round 3 (3 Questions, 5 Minutes)

1. A Friedman number is one which is equal to an expression using each of its digits along with one or more uses of addition, subtraction, multiplication, division, and exponentials (using parenthesis is allowed). The following are Friedman numbers:

$$25 = 5^2$$

$$216 = 6^{2+1}$$

$$1022 = 2^{10} - 2$$

$$1024 = (4 - 2)^{10}$$

One of 288, 289, and 290 is a Friedman number. Which one?

2. If $x = -1$ is a root of

$$p(x) = ax^5 - 4x^4 + 7x^3 - 10x^2 + 11x + 90,$$

what is a ?

3. How many distinct rearrangements are there of the letters SECRETS?

Lunch!

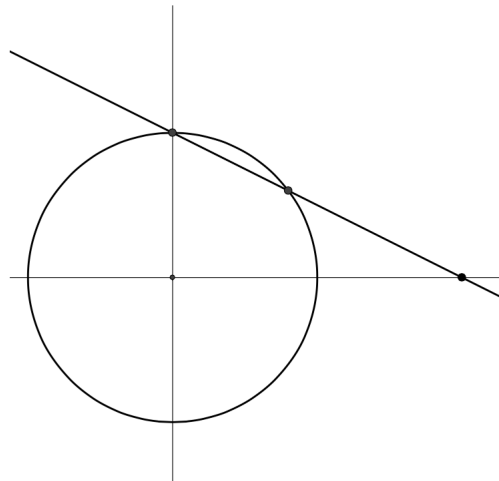
Stage 2

Stage 2, Round 1 (Blitz Round, 3 Minutes)

- a. At a dinner party with three couples, everyone shakes hands with everybody else except their own spouse. How many handshakes occur?
- b. If length of a rectangle doubles and the width shrinks by a factor of one quarter, the new rectangle's area is what multiple of the old rectangle's area?
- c. What is 2017 in binary (base 2)?
- d. Consider the sequence $a_1 = 2, a_2 = 7, a_3 = 8, a_4 = 13, a_5 = 14, a_6 = 19, \dots$.
If you continue this sequence, what is a_{10} ?
- e. If they are made out of stretchable, squishy clay, which of the following two shapes can be deformed from one to the other without gluing and without making any cuts, holes, punctures, etc.? Shapes: a coffee mug, a bowl, a bagel, and a ladder.

Stage 2, Round 2 (3 Questions, 5 Minutes)

1. If $a_1 = 1$, $a_2 = 1$, and $a_{n+1} = a_{n-1} - 2a_n$ for $n \geq 2$, then is a_{2017} even or odd?
2. In the figure below is the unit circle centered at the origin. The straight line goes through the point $(0, 1)$ (the “north pole” of the circle) and $(2, 0)$ on the x -axis. What is the x -coordinate of the other point where the circle and line intersect?



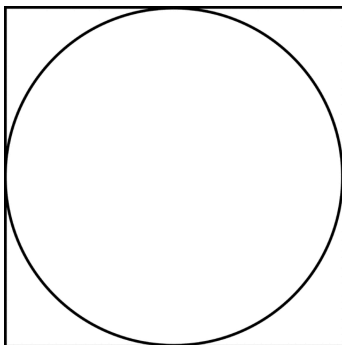
3. How many ordered pairs of positive integers (a, b) satisfy $a + b \leq 10$?

Stage 3

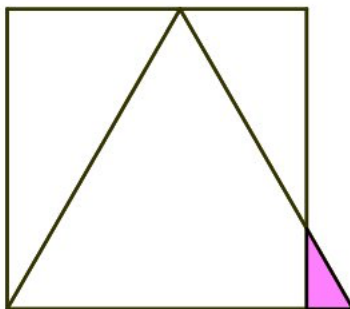
Stage 3, Round 1 (3 Questions, 5 Minutes)

1. In the following image, what is the ratio

$$\frac{\text{Area of Square}}{\text{Area of Circle}}?$$

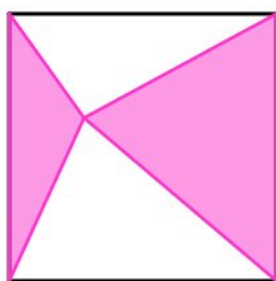


2. Which is larger, $100!$ or 10^{100} ?
3. If the largest triangle in the image is an equilateral triangle with side length 2, then what is the area of the square?



Stage 3, Round 2 (2 Questions, 5 Minutes)

1. In the following image, what fraction is shaded? Please explain your reasoning!



2. Let $p(x)$ be the degree four polynomial which has the first 4 prime numbers as roots and has leading coefficient equal to 1. What is the constant term of $p(x)$?
3. How many diamonds are in the following picture?



The End!

Spot Prize I (Word Search!)

Name: _____

School: _____

R D F O J F Q E M G O L X N E M O R D N I L A P
 N O G A X E L F A X E H E L Z Z U P P D R J K K
 D H N D O L Y R W I A R R J P U C O Q I W R A U
 E P Y O O L E N I G M A O X R Y X I V E D B G C
 A T L R I N M M J F R J K Z O D S D R A C I T V
 O D R N D T I N B A N E J M B E L F F U H S P B
 A A J R H V A N D I E R E N A W M Y E J L U U P
 C N A P W P R T A Z S B Y S B G M G L A U R Y O
 S G Y E V O K W U B C D O N I A I O C C E C W M
 T M T M N L D P G M F M U T L Q K C R J N R T O
 E K R M O Y H J V J R F X A I E H N I H N Y T N
 R P D Y K N H S I W F E T X T M K Q C Q V P K I
 C I F Q L O H W R Q L N P Q Y D K X O P J T V C
 E G B S A M T T M C U W G F T O Y K D H D O G A
 S Z Y I H I P W A K B K H U V Y M O R R I S U P
 Y D V W O A I P A M O Z L A Y Z I C M S B R L K
 A C N E M L V H N P I S L A Q K D L A O N E V H
 W T I L A N S A Q Y A E S M W E P N T N A H P H
 W C T X K O Y C D K V H P V O N P I R I W T B O
 Q C R Q G R X R T S Q U A R E N G P I A R E S A
 Z K A P Y M K E K F Q I F L U A Z S X E J O Y A
 O O M Q H A K Y D C L L E V R R F L B O G N Q P
 D P O E S N Y Y A O Z I E L X D I O A Z I Z O E
 M G S E A R O T N A C F J H B N R V U N D O V P

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|--------------|----------------|---------------|--------------|
| • MONICA | • DEVI | • CARDS | • OKLAHOMA |
| • VANDIEREN | • HEXAFLEXAGON | • PERMUTATION | • POLYNOMIAL |
| • ROBERT | • DR MATRIX | • MAGIC | • ENIGMA |
| • MORRIS | • CANTOR | • PROBABILITY | • PALINDROME |
| • LEWIS | • EMMY | • CRYPTO | • SQUARE |
| • CARROLL | • NOETHER | • MATH | • CIRCLE |
| • SECRETS | • MARTIN | • PUZZLE | • CODE |
| • KENNARD | • GARDNER | • TULSA | • SONIA |
| • SHAKUNTALA | • SHUFFLE | • NORMAN | • KOVALEVSKY |

Spot Prize I (Word Search!)

Name: _____

School: _____

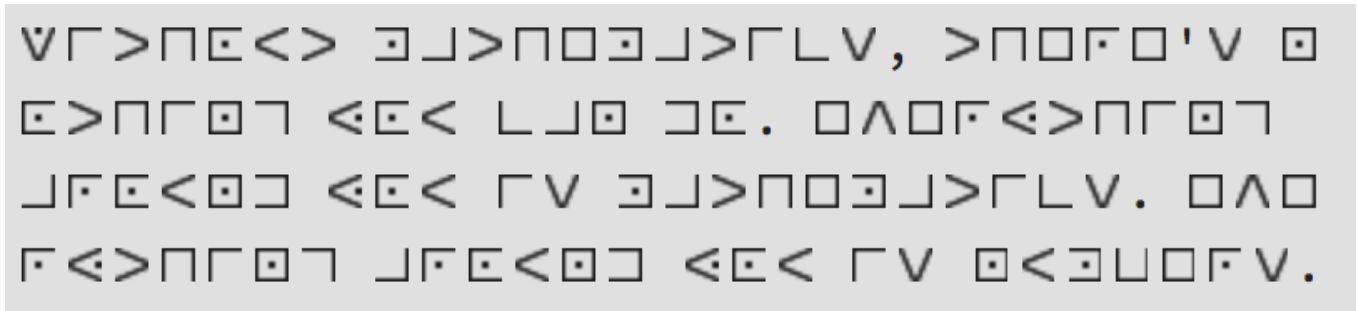
R	D	F	O	J	F	Q	E	M	G	O	L	X	N	E	M	O	R	D	N	I	L	A	P
N	O	G	A	X	E	L	F	A	X	E	H	E	L	Z	Z	U	P	P	D	R	J	K	K
D	H	N	D	O	L	Y	R	W	I	A	R	R	J	P	U	C	O	Q	I	W	R	A	U
E	P	Y	O	O	L	E	N	I	G	M	A	O	X	R	Y	X	I	V	E	D	B	G	C
A	T	L	R	I	N	M	M	J	F	R	J	K	Z	O	D	S	D	R	A	C	I	T	V
O	D	R	N	D	T	I	N	B	A	N	E	J	M	B	E	L	F	F	U	H	S	P	B
A	A	J	R	H	V	A	N	D	I	E	R	E	N	A	W	M	Y	E	J	L	U	U	P
C	N	A	P	W	P	R	T	A	Z	S	B	Y	S	B	G	M	G	L	A	U	R	Y	O
S	G	Y	E	V	O	K	W	U	B	C	D	O	N	I	A	I	O	C	C	E	C	W	M
T	M	T	M	N	L	D	P	G	M	F	M	U	T	L	Q	K	C	R	J	N	R	T	O
E	K	R	M	O	Y	H	J	V	J	R	F	X	A	I	E	H	N	I	H	N	Y	T	N
R	P	D	Y	K	N	H	S	I	W	F	E	T	X	T	M	K	Q	C	Q	V	P	K	I
C	I	F	Q	L	O	H	W	R	Q	L	N	P	Q	Y	D	K	X	O	P	J	T	V	C
E	G	B	S	A	M	T	T	M	C	U	W	G	F	T	O	Y	K	D	H	D	O	G	A
S	Z	Y	I	H	I	P	W	A	K	B	K	H	U	V	Y	M	O	R	R	I	S	U	P
Y	D	V	W	O	A	I	P	A	M	O	Z	L	A	Y	Z	I	C	M	S	B	R	L	K
A	C	N	E	M	L	V	H	N	P	I	S	L	A	Q	K	D	L	A	O	N	E	V	H
W	T	I	L	A	N	S	A	Q	Y	A	E	S	M	W	E	P	N	T	N	A	H	P	H
W	C	T	X	K	O	Y	C	D	K	V	H	P	V	O	N	P	I	R	I	W	T	B	O
Q	C	R	Q	G	R	X	R	T	S	Q	U	A	R	E	N	G	P	I	A	R	E	S	A
Z	K	A	P	Y	M	K	E	K	F	Q	I	F	L	U	A	Z	S	X	E	J	O	Y	A
O	O	M	Q	H	A	K	Y	D	C	L	L	E	V	R	R	F	L	B	O	G	N	Q	P
D	P	O	E	S	N	Y	Y	A	O	Z	I	E	L	X	D	I	O	A	Z	I	Z	O	E
M	G	S	E	A	R	O	T	N	A	C	F	J	H	B	N	R	V	U	N	D	O	V	P

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Spot Prize II (Break the Code!)

Name: _____ School: _____

Late at night you and a friend sneak onto the property of the Oklahoma Spaceport. Inside a warehouse you find a stone tablet with the following carved into it:

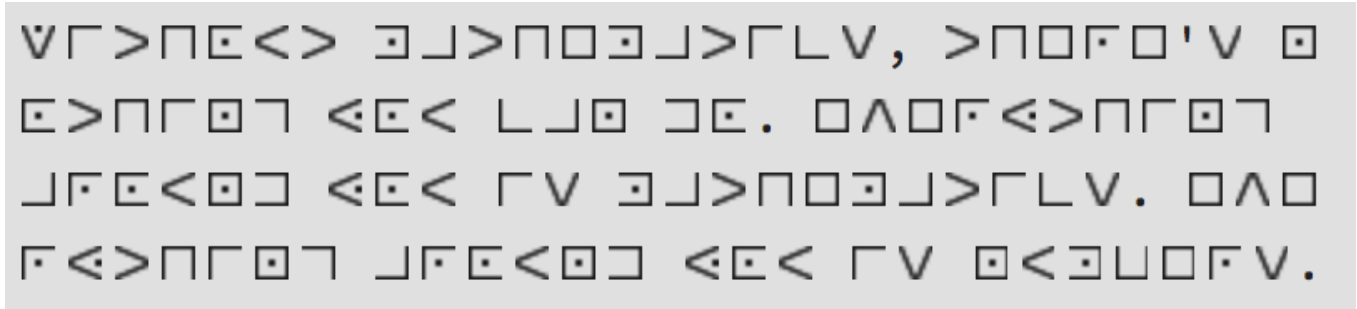


Your friend thinks this must be an alien code where each symbol stands for a letter of the alphabet. Fortunately your friend is also part alien and is able to tell you that □ is “E” and > is “T”. Can you crack the code before time runs out?

Spot Prize II (Break the Code!)

Name: _____ School: _____

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Lunch Problem

Name: _____ School: _____

Due after lunch at the door to the Math Bowl.

Write your solution on the back.

While at the Oklahoma Spaceport, you discover what looks to be an alien document describing their invasion plan. NASA knows how to translate their language not their number system. Unlike the ten figures humans have, aliens only have five fingers. This means their number system only has five digits and NASA is stuck. The aliens use the following symbols (helpfully translated by your half-alien friends, Joey and Erin¹). The first four represent the numbers 0, 1, 2, 3, 4 and are called *regular symbols*. There is also the *special symbol* which represents 5.

$$\odot = 0 \quad \uplus = 1 \quad \not\sim = 2 \quad \propto = 3 \quad \Xi = 4 \quad \forall = 5$$

When the aliens write a number, they write in the form _____ $\forall$ _____ where there is a single regular symbol to the right of $\forall$ and there is a string of symbols (both regular and special) to the left. The string on the left determines how many fives go into the number. You add the number given by the symbol on the right to get the value of the number. Here are some simple examples:

$$\propto \forall \uplus = 3 \cdot 5 + 1 = 16$$

$$\odot \forall \not\sim = 0 \cdot 5 + 2 = 2$$

$$\uplus \forall \odot = 1 \cdot 5 + 0 = 5.$$

Notice that a symbol is not used by itself to represent a number because this wouldn't be in the correct format. Here is a more complicated example. You read right to left using the format repeatedly if needed to decode the number and we use parenthesis showing the order in which decoding is done:

$$\uplus \forall \odot \forall \odot = (\uplus \forall \odot) \forall (\odot) = (1(5) + 0)5 + 0 = 25$$

¹Thanks to Joey Randich and Erin Hausmann for sharing their knowledge!

Another example:

$$\begin{aligned}
 \not\sim \forall \oplus \forall \not\sim \forall \oplus &= (\not\sim \forall \oplus \forall \not\sim) \forall (\oplus) \\
 &= (\not\sim \forall \oplus \forall \not\sim) 5 + 1 \\
 &= ((\not\sim \forall \oplus) \forall (\not\sim)) 5 + 1 \\
 &= ((2 \cdot 5 + 1) \cdot 5 + 2) 5 + 1 \\
 &= 286.
 \end{aligned}$$

Here is the alien document. Can you translate the numbers for NASA?

Invasion Plan (Top Secret!)

Number of command ships: $\not\sim \forall \not\sim \forall \Xi =$ _____.

Number of troop ships: $\propto \forall \oplus \forall \Xi \forall \propto \forall \oplus \forall \not\sim \forall \odot =$ _____.

Number of supply ships: $\not\sim \forall \oplus \forall \oplus \forall \Xi =$ _____.

Longitude: $- \propto \forall \Xi \forall \not\sim . \oplus \forall \odot \forall \oplus \forall \not\sim \forall \propto \forall \propto \forall \Xi \forall \Xi =$ _____.

Latitude: $\oplus \forall \not\sim \forall \odot . \not\sim \forall \oplus \forall \oplus \forall \oplus \forall \Xi \forall \odot \forall \not\sim =$ _____.

Invasion date: $\not\sim \forall \oplus / \propto \forall \not\sim / \propto \forall \oplus \forall \odot \forall \not\sim \forall \not\sim =$ _____.