

Name:

1. Find the limit or show that the limit does not exist.

a. (10 points)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + y^3 + 7}{x + y + 1}$$

b. (10 points)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y}{x^4 + y^2}$$

2. (10 points) Find the partial derivatives f_x , f_y , and f_z for $f(x, y, z) = yz \ln(xy)$.

3. (10 points) Use the chain rule to find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial \theta}$ when $r = 2$ and $\theta = \frac{\pi}{2}$ for the following function:

$$w = xy + yz + zx, \quad x = r \cos \theta, \quad y = r \sin \theta, \quad z = r\theta.$$

4. (10 points) Suppose a surface S is defined by the equation $x^3 + y^2 + z^3 + xy = 4$. Find an equation for the tangent plane at the point $(1, 1, 1)$.

5. (10 points) Find the derivative of the function $f(x, y) = xy + y^2$ in the direction of $\mathbf{v} = \langle 1, 1 \rangle$ at the point $(3, 2)$.

6. (10 points) Find the local maxima, local minima, and saddle points of the function $f(x, y) = x^3 + y^3 + 3x^2 - 3y^2 - 8$.

7. (10 points) Find the maximum and minimum values of $x^2 + y^2$ subject to the constraint $x^2 - 2x + y^2 - 4y = 0$.

(Extra page for #7)