4.5.2 Let $p = P$(missile scores direct hit) = 0.30. Then P(target will be destroyed by seventh missile fired) = P(exactly three direct hits occur among first six missiles <u>and</u> seventh missile scores direct hit) = $\binom{6}{3}(0.30)^3(0.70)^3(0.30) = 0.056$.

4.6.4 If $E(Y) = \dfrac{r}{\lambda} = 1.5$ and $\text{Var}(Y) = \dfrac{r}{\lambda^2} = 0.75$, then $r = 3$ and $\lambda = 2$, which makes $f_Y(y) = 4y^2e^{-2y}$, $y > 0$. Then $P(1.0 \le Y_i \le 2.5) = \int_{1.0}^{2.5} 4y^2e^{-2y}\,dy = 0.55$. Let X = number of Y_i's in the interval $(1.0, 2.5)$. Since X is a binomial random variable with $n = 100$ and $p = 0.55$, $E(X) = np = 55$.

4.6.10 $M_Y(t) = (1 - t/\lambda)^{-r}$ so

$$M_Y^{(1)}(t) = \frac{d}{dt}(1-t/\lambda)^{-r} = r(1-t/\lambda)^{-r-1}(-1/\lambda) = \frac{r}{\lambda}(1-t/\lambda)^{-r-1}$$

and

$$M_Y^{(2)}(t) = \frac{d}{dt}\frac{r}{\lambda}(1-t/\lambda)^{-r-1} = \frac{r}{\lambda}(-r-1)(1-t/\lambda)^{-r-2}(-1/\lambda) = \frac{r(r+1)}{\lambda}(1-t/\lambda)^{-r-2}$$

For an arbitrary integer $m \geq 2$, we can generalize the above to see that

$$M_Y^{(m)}(t) = \frac{r(r+1)...(r+m-1)}{\lambda^m}(1-t/\lambda)^{-r-m}.$$

Then $E(Y^m) = M_Y^{(m)}(0) = \frac{r(r+1)...(r+m-1)}{\lambda^m}$.

But note that $\frac{r(r+1)...(r+m-1)}{\lambda^m} = \frac{\Gamma(r+m)}{\Gamma(r)\lambda^m}$

The right hand side of the equation is equal to the expression in Question 4.6.8 when r is an integer.