**3.5.18**

| Outcome | $X$ |
|---|---|
| HHH | 6 |
| HHT | 2 |
| HTH | 4 |
| HTT | 1 |
| THH | 2 |
| THT | 0 |
| TTH | 1 |
| TTT | 0 |

From the table, we can calculate $p_X(0) = 1/4$, $p_X(1) = 1/4$, $p_X(2) = 1/4$, $p_X(4) = 1/8$, $p_X(6) = 1/8$.
Then $E(X) = 0 \cdot (1/4) + 1 \cdot (1/4) + 2 \cdot (1/4) + 4 \cdot (1/8) + 6 \cdot (1/8) = 2$

**3.5.8** (a) $E(Y)=\int_0^1 y\cdot 3(1-y)^2dy=\int_0^1 3(y-2y^2+y^3)dy$

$$=3\left[\frac{1}{2}y^2-\frac{2}{3}y^3+\frac{1}{4}y^4\right]_0^1=\frac{1}{4}$$

(b) $E(Y)=\int_0^\infty y\cdot 4ye^{-2y}dy=4\left[-\frac{1}{2}y^2e^{-2y}-\frac{1}{2}ye^{-2y}-\frac{1}{4}e^{-2y}\right]_0^\infty=1$

(c) $E(Y)=\int_0^1 y\cdot\left(\frac{3}{4}\right)dy+\int_2^3 y\cdot\left(\frac{1}{4}\right)dy=\left.\frac{3y^2}{8}\right|_0^1+\left.\frac{y^2}{8}\right|_2^3=1$

(d) $E(Y) = \int_0^{\pi/2} y \cdot \sin y \, dy = (-y\cos y + \sin y) \Big|_0^{\pi/2} = 1$

**3.6.2** $\mu = \int_0^1 y\left(\frac{3}{4}\right)dy + \int_2^3 y\left(\frac{1}{4}\right)dy = 1$

$$E(X^2) = \int_0^1 y^2\left(\frac{3}{4}\right)dy + \int_2^3 y^2\left(\frac{1}{4}\right)dy = \frac{11}{6}$$

$$\text{Var}(X) = \frac{11}{6} - 1 = \frac{5}{6}$$