3.7.22 $f_Y(y) = \int_0^y 2e^{-x}e^{-y}dx = -2e^{-x}e^{-y}\Big|_0^y = 2e^{-y} - 2e^{-2y},\ 0 \le y$

3.7.28 (a) $$F_{X,Y}(u, v) = \int_0^u \int_x^v \frac{1}{2} dy\, dx = \int_0^u \left[\frac{1}{2} y \Big|_0^v \right] dx = \int_0^u \frac{1}{2}(v - x) dx = \frac{1}{2} uv = \frac{1}{4} u^2$$

(b) $$F_{X,Y}(u, v) = \int_0^v \int_y^u \frac{1}{x} dy\, dx = \int_0^v \left[\ln x \Big|_y^u \right] dy = \int_0^v \ln u - \ln y dy = v \ln u - v \ln v + v$$

(c) Case I: $v \le 1 - u$

$$F_{X,Y}(u, v) = \int_0^v \int_0^u 6x dx\, dy = \int_0^v \left[3x^2 \Big|_0^u \right] dy = \int_0^v 3u^2 dy = 3u^2 v$$

Case II: $v > 1 - u$

$$F_{X,Y}(u, v) = \int_0^u \int_0^v 6x dy\, dx = \int_{1-v}^u \int_{1-x}^v 6x dy\, dx$$

$$= 3u^2v - [3u^2v - 3u^2 + 2u^3 - 3(1 - v)^2 v + 3(1 - v)^2 - 2(1 - v)^3]$$

$$= 3u^2 - 2u^3 + 3(1 - v)^2 v - 3(1 - v)^2 + 2(1 - v)^3 = 3u^2 - 2u^3 - (1 - v)^3$$

3.7.30 By Theorem, 3.7.3, $f_{X,Y} = \dfrac{\partial^2}{\partial x \partial y} F_{X,Y} = \dfrac{\partial^2}{\partial x \partial y}[(1-e^{-\lambda y})(1-e^{-\lambda x})]$

$$= \frac{\partial}{\partial x}\frac{\partial}{\partial y}[(1-e^{-\lambda y})(1-e^{-\lambda x})] = \frac{\partial}{\partial x}[(\lambda e^{-\lambda y})(1-e^{-\lambda x})]$$

$$= \lambda e^{-\lambda y}\lambda e^{-\lambda x}, x \geq 0, y \geq 0$$

3.7.8

Outcome	X	Y
H H H	1	3
H H T	0	2
H T H	1	2
H T T	0	1
T H H	1	2
T H T	0	1
T T H	1	1
T T T	0	0

(x, y)	$p_{X,Y}(x, y)$
(0, 0)	1/8
(0, 1)	2/8
(0, 2)	1/8
(0, 3)	0
(1, 0)	0
(1, 1)	1/8
(1, 2)	2/8
(1, 3)	1/8