

$$3.7.44 \quad F_X(x) = \int_0^x \frac{t}{2} dt = \frac{x^2}{4}. \quad F_Y(y) = \int_0^y 2t dt = y^2$$

$$F_{X,Y}(x, y) = F_X(x)F_Y(y) = \frac{x^2 y^2}{4}, \quad 0 \leq x \leq 2, \quad 0 \leq y \leq 1$$

$$3.8.2 \quad f_{X+Y}(w) = \int_{-\infty}^{\infty} f_X(x) f_Y(w-x) dx = \int_0^w (xe^{-x})(e^{-(w-x)}) dx = e^{-w} \int_0^w x dx = \frac{w^2}{2} e^{-w}, \quad w \geq 0$$

3.9.14 From Question 3.9.3, we have $E(X + Y) = E(X) + E(Y) = 5/9 + 11/18 = 21/18 = 7/6$

$$\begin{aligned} E[(X + Y)^2] &= \int_0^1 \int_0^1 (x + y)^2 \frac{2}{3}(x + 2y) dx dy \\ &= \frac{2}{3} \int_0^1 \int_0^1 (x^2 + 2xy + y^2)(x + 2y) dx dy = \frac{2}{3} \int_0^1 \int_0^1 (x^3 + 2x^2y + xy^2 + 2x^2y + 4xy^2 + 2y^3) dx dy \\ &= \frac{2}{3} \int_0^1 \int_0^1 (x^3 + 4x^2y + 5xy^2 + 2y^3) dx dy = \frac{2}{3} \int_0^1 \left(\frac{1}{4}x^4 + \frac{4}{3}x^3y + \frac{5}{2}x^2y^2 + 2xy^3 \right) \Big|_0^1 dy \\ &= \frac{2}{3} \int_0^1 \left(\frac{1}{4} + \frac{4}{3}y + \frac{5}{2}y^2 + 2y^3 \right) dy = \frac{2}{3} \left(\frac{1}{4} + \frac{4}{6} + \frac{5}{6} + \frac{2}{4} \right) = \frac{3}{2} \end{aligned}$$

$$\text{Then } \text{Var}(X + Y) = E[(X + Y)^2] - E(X + Y)^2 = \frac{3}{2} - \left(\frac{7}{6} \right)^2 = \frac{5}{36}$$

3.9.8 Let X_1 = number showing on face 1; X_2 = number showing on face 2. Since X_1 and X_2 are independent, $E(X_1X_2) = E(X_1)E(X_2) = (3.5)(3.5) = 12.25$.