Assignment #4 2-22-05

p 78 5.1, 5.3

(Use the generator matrix for Ham(4) on p 62):

5.2 When $(a\ b\ c\ d\ e\ f\ g\ h\ i\ j\ k)^T$ is multiplied by the generator matrix, rows 1 through 11 give the entries $a, b, c, \ldots, j, k$ and rows 12, 13, 14, 15 give

$$\left.\begin{aligned} u &= a+d+e+g+h+i+j+k \\ x &= a+b+c+d+f+g+h+i \\ y &= b+c+e+g+h+i+j \\ z &= c+d+f+h+i+j+k \end{aligned}\right\} (*)$$

In the following table, there is a "check" if the variable $a \ldots k$ appears in $u \ldots z$:

	a	b	c	d	e	f	g	h	i	j	k
u	✓			✓		✓	✓	✓	✓	✓	✓
x	✓	✓	✓	✓		✓	✓	✓	✓		
y		✓	✓		✓		✓	✓	✓	✓	
z			✓	✓		✓		✓	✓	✓	✓

Notice that every column of the table has a different pattern of checks. If the element of the top of the column is changed, then the element among $u x y z$ that change are those checked. Since all columns are different, any single error in $a \ldots k$ changes $u x y z$ differently. And any single error among $u x y z$ changes $u x y z$ differently as well. Thus any single error changes a different set of the relations $(*)$, errors in $a \ldots k$ changing at least 2, errors in $u x y z$ changing only one.

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5.3 (Refer to Theorem p. 19) If $d(H') = d(K') = 4$, then H' & K' can correct t errors and detect $s+t$ errors if $d(H') \geq 2s+t+1$, $d(K') \geq 2s+t+1$, and $4 = 2\cdot 1 + 1 + 1$ (i.e. $s = t = 1$), so will correct 1, detect 2. By examining the codewords on page 7, we see $d(H') = d(K') = 4$.

Now suppose $u \in H'$ and $wt(u) = 4$. Then $\exists\ v \in \mathbb{Z}_2^8$ such that $d(u, v) = 2 = d(v, 0)$. $(= wt(v))$. Then if v is received, it differs by 2 errors from 2 codewords, u and 0. So we cannot correct 2 errors made in u, in general. Same reasoning applies in ~~H~~ H'.