Assignment # 7 Due 4-5-05

$f = x^3 + x + 1 \qquad GF(2^3) = \mathbb{Z}_2(x)/f$

$= \{0, 1, x, x+1, x^2, x^2+1, x^2+x, x^2+x+1\}$

$\alpha = x + 1$

Write $H_{3,2}$ as powers of α, and in binary.
Determine rank of BCH(3,2)

$\alpha^1 = x+1 = 011 \qquad \alpha^2 = x^2+1 = 101 \qquad \alpha^3 = x^2 = 010$
$\alpha^4 = x^2+x+1 = 111 \qquad \alpha^5 = x = 001 \qquad \alpha^6 = x^2+x = 110$
$\alpha^7 = 1 = 001$

$$V_{3,2} = \left[\alpha^{(n-i)2j}\right] = \begin{bmatrix} \alpha^6 & \alpha^5 & \alpha^4 & \alpha^3 & \alpha^2 & \alpha & 1 \\ \alpha^{12} & \alpha^{10} & \alpha^8 & \alpha^6 & \alpha^4 & \alpha^1 & 1 \\ \alpha^{18} & \alpha^{15} & \alpha^{12} & \alpha^9 & \alpha^6 & \alpha^3 & 1 \\ \alpha^{24} & \alpha^{20} & \alpha^{16} & \alpha^{12} & \alpha^8 & \alpha^4 & 1 \end{bmatrix}$$

$$\boxed{H_{3,2} = \begin{bmatrix} \alpha^6 & \alpha^5 & \alpha^4 & \alpha^3 & \alpha^2 & \alpha & 1 \\ \alpha^4 & \alpha & \alpha^5 & \alpha^2 & \alpha^6 & \alpha^3 & 1 \end{bmatrix}}$$

In binary

$$\boxed{H_{3,2} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}}$$

Suppose $a = (a_6\, a_5\, a_4\, a_3\, a_2\, a_1\, a_0) \in BCH(3,2)$
Then $H_{3,2}\, a^T = \underline{0}$ or

(1) $a_6 + a_4 + a_2 = 0$
(2) $a_6 + a_4 + a_3 + a_1 = 0$
(3) $a_5 + a_4 + a_2 + a_1 + a_0 = 0$
(4) $a_6 + a_3 + a_2 = 0$
(5) $a_6 + a_5 + a_2 + a_1 = 0$
(6) $a_6 + a_5 + a_4 + a_0 = 0$

Add equation (1) to (2), (4), (5), (6):

$a_6 + a_4 + a_2 = 0$
$a_3 + a_2 + a_1 = 0$
$a_5 + a_4 + a_2 + a_1 + a_0 = 0$
$a_4 + a_3 = 0 \implies a_4 = a_3$
$a_5 + a_4 + a_1 = 0$
$a_5 + a_2 + a_0 = 0$

Substitute $a_4 = a_3$ and drop equation (4)

$a_6 + a_3 + a_2 = 0$
$a_3 + a_2 + a_1 = 0$
$a_5 + a_3 + a_2 + a_1 + a_0 = 0$
$a_5 + a_3 + a_1 = 0$
$a_5 + a_2 + a_0 = 0$

Plug equation (2) into equation (3) to get $a_5 + a_0 = 0$ or $a_5 = a_0$
Substitute $a_5 = a_0$ and drop equation (3)

$a_6 + a_3 + a_2 = 0$
$a_3 + a_2 + a_1 = 0$
$a_3 + a_1 + a_0 = 0$
$a_2 = 0$

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Since $a_2=0$, ① becomes $a_6=a_3$ and ② becomes $a_3=a_1$.
Then ③ becomes $a_0=0$.
Thus ~~$a_3=a_4$~~ $a_6=a_3=a_1=a_4$ and $a_5=a_2=a_0=0$
So $a=(1011010)$ or $a=(0000000)$
$|DCH(3,2)|=2$ rank $\boxed{DCH(3,2)=1}$