Announcements:

- Course Discussion Sections

  Dustin Gaskins : 014, 015

  Ryan Reynolds : 011, 013

  Noah Torgenson : 012

- First WebWork assignment due by Sunday January 31, 11 PM ....
- Class notes and more posted on course web site...
- Office Hours and Math Center...

Functions of One Variable

Let D be a set of real numbers.

A function $f(x)$ with domain D is a rule that assigns a real number $f(x)$ to each real number x in D.

- domain$(f) = D$ = set of "inputs" for $f(x)$.
- The range of f is the set of all numbers $f(x)$ where x is in D.

  range(f) = set of "outputs" for $f(x)$
- The graph of $f(x)$ is the set of all points $(x, f(x))$ in the xy-plane where x is in D.

We say the graph of $f(x)$ is the graph of the equation $y = f(x)$

Comments:

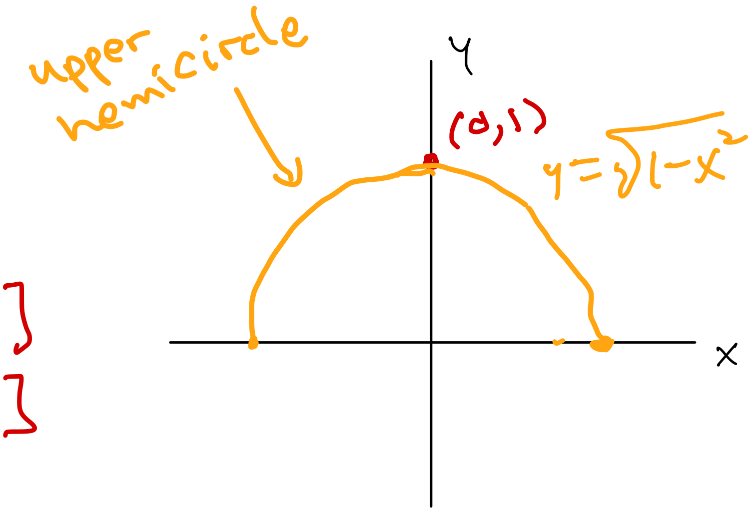
① domain is very important!!

② range(f) can be difficult to determine.

③ The graph of f is usually a "curve" in the xy-plane.

examples

① $f(x) = \sqrt{1-x^2}$



$\text{domain}(f) = [-1, 1]$

$\text{range}(f) = [0, 1]$

$\sqrt{1-x^2}$ only makes sense if $1-x^2 \geq 0$

$f(0) = \sqrt{1-0} = 1$

$y = \sqrt{1-x^2}$ square

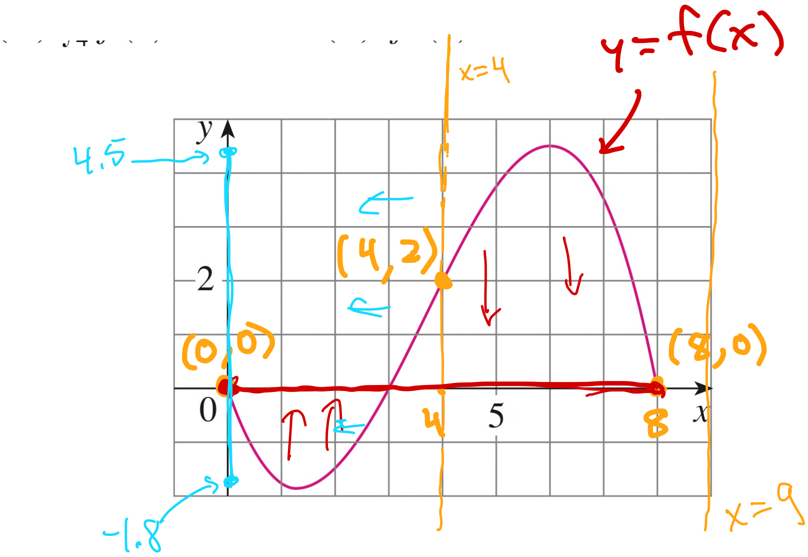
$y \geq 0 \quad y^2 = 1 - x^2$

$x^2 + y^2 = 1$ circle

② $g(t) = \sqrt{1-t^2}$. This is exactly the same function as ① !

"dummy variable"

③



$\text{domain}(f) =$

$\text{range}(f) =$

$f(0) = 0$

$f(8) = 0$

$f(6) = 4.5$

$f(4) = 2$

The graph of a function satisfies the vertical line property (VLP).

9 is not in domain(f). ← $f(9) = \text{DNE}$

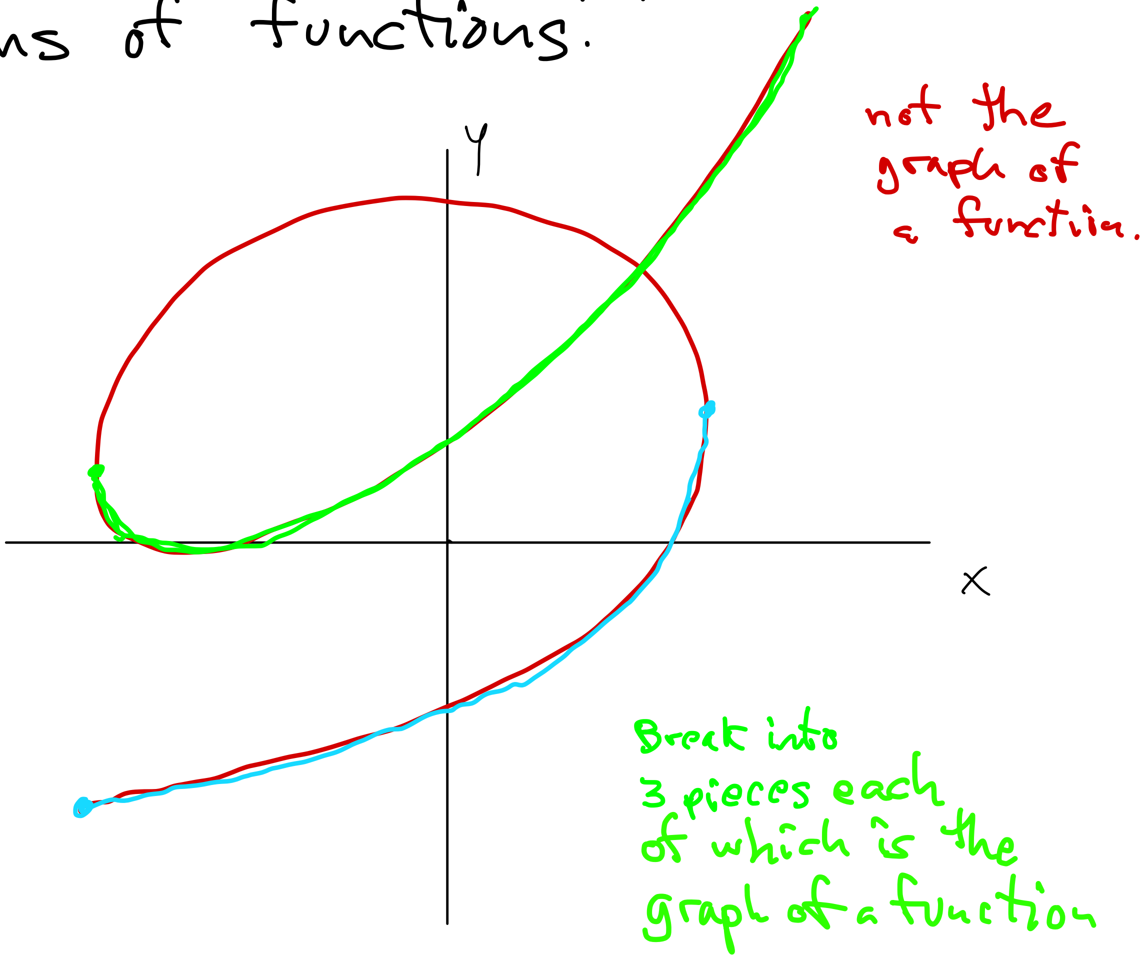
4 is in domain(f).

$\text{domain}(f) = [0, 8]$

$\text{range}(f) = [-1.8, 4.5]$

④

Many curves in the xy-plane are not graphs of functions.

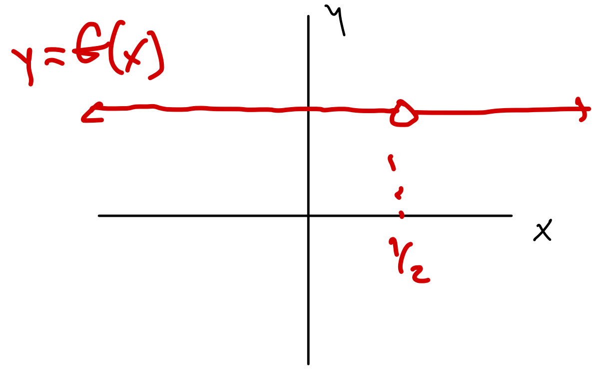


(Arbitrary curves can be viewed as the trace of a particle in motion. We won't examine this perspective in Calc 2 but it becomes very important in Calc 3 and Calc 4.)

5. $$G(x) = \frac{1-2x}{1-2x}$$

$\text{domain}(G) = \mathbb{R} \sim \{1/2\}$

Because $G(1/2) = \text{DNE}$



$G(1/2) = \text{DNE}$

$G(x) = 1\ ,\ x \neq 1/2$

piecewise function

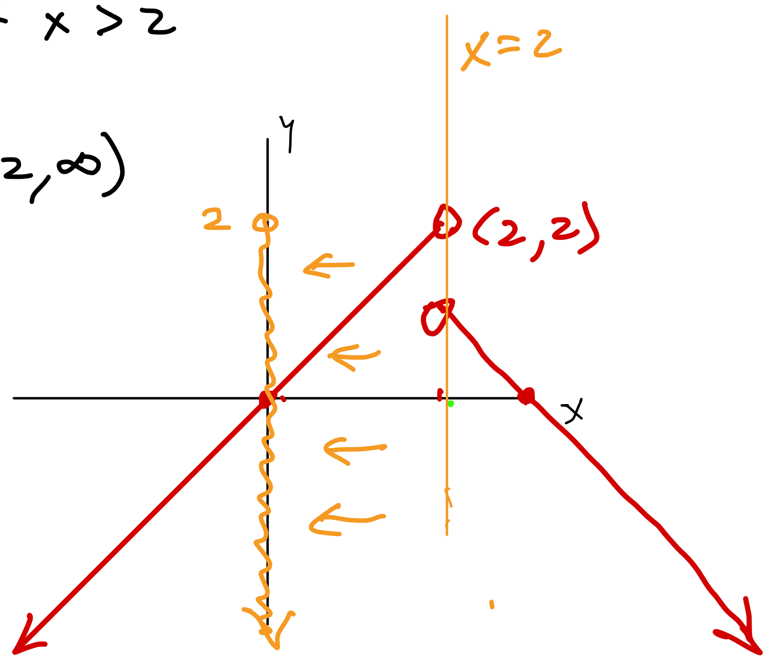
$h(3) = 3-3 = 0$

$h(0) = 0$

6. $$h(x) = \begin{cases} x & \text{if } x < 2 \\ 3-x & \text{if } x > 2 \end{cases}$$

$\text{domain}(h) = \mathbb{R} - \{2\} = (-\infty, 2) \cup (2, \infty)$

$\text{range}(h) = (-\infty, 2)$



$h(2) = \text{DNE}$

⑦ Find the range of the polynomial function

$$f(x) = x^4 - 132x^3 - 200x^2$$

degree 4 polynomial.

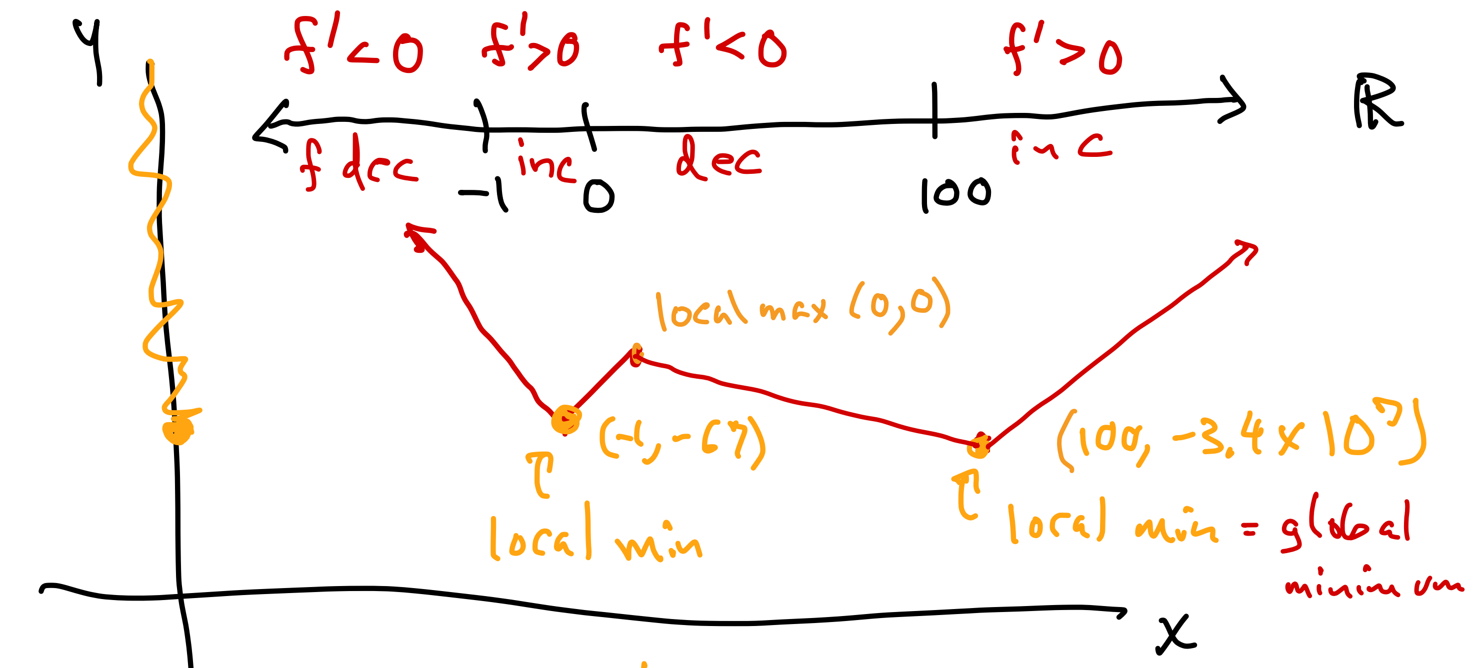
$\text{domain}(f) = \mathbb{R} = (-\infty, \infty) = (-\text{inf}, \text{inf})$

Let's sketch the graph.

$$\begin{aligned} f'(x) &= 4x^3 - 396x^2 - 400x \\ &= 4x(x^2 - 99x - 100) \\ &= 4x(x+1)(x-100) \end{aligned}$$

So $f'(x) = 0$ when $x = 0$, $x = -1$ or $x = 100$.

So $0, -1, 100$ are critical points for f

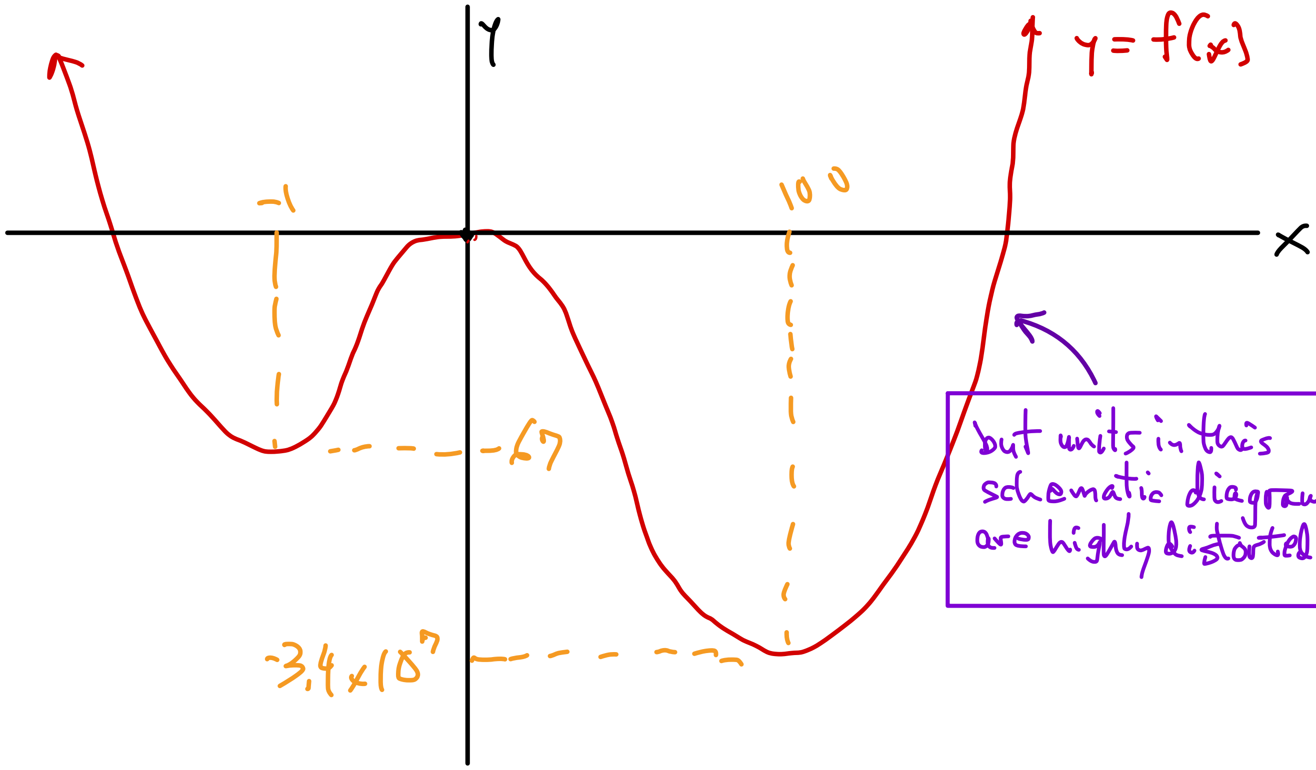


x	f(x)
-1	-67
100	~3.4 x 10^7
0	0

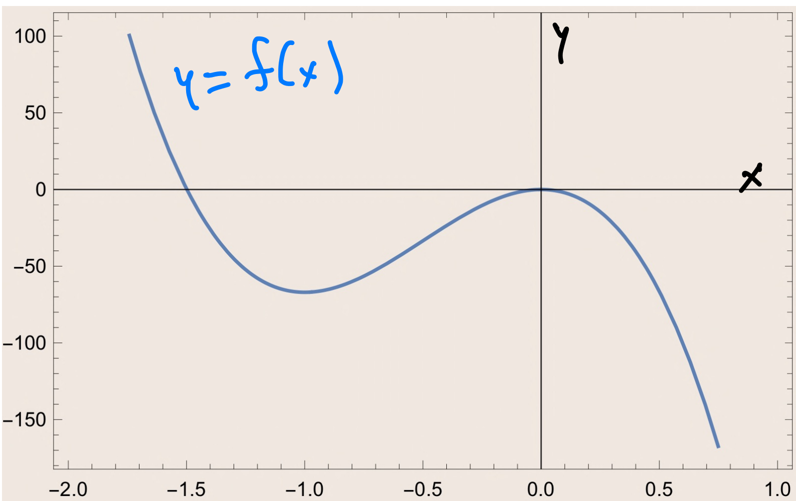
note: f(100) actually equals 3.4 x 10^7

$\text{range}(f)$
$= (f(100), \infty)$
$= (-3.4 \times 10^7, \infty)$

With concavity information the graph of $f(x) = x^4 - 132x^3 - 200x^2$ looks something like:

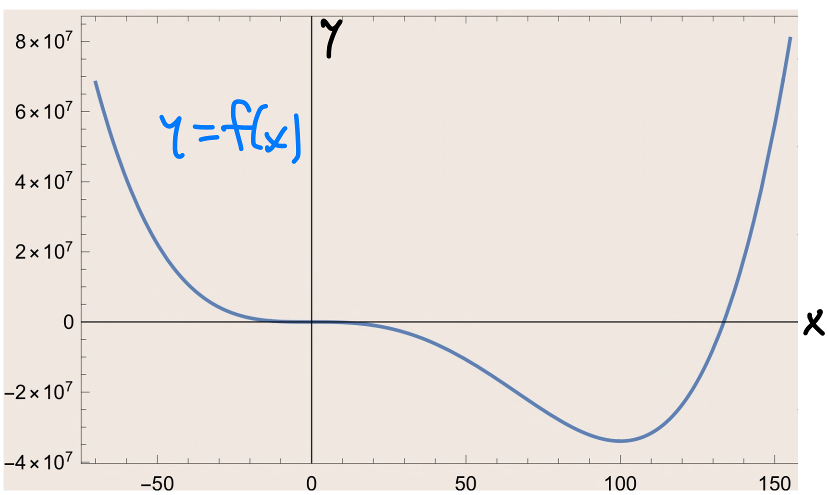


but units in this schematic diagram are highly distorted!



Here's a graphing calculator picture with correct units and $-2 \le x \le 1$

Here's a graphing calculator picture with correct units and $-70 \le x \le 155$



In practice no single window can show all important features of the graph of $y = f(x)$.