Notation for antiderivatives

no limits of integration!

Write $\int f(x)\,dx$ to denote the most general antiderivative of $f(x)$.

e.g. $\int x\,dx = \frac{1}{2}x^2 + C$

We call $\int f(x)\,dx$ the <u>indefinite integral of $f(x)$ with respect to x</u>

Stewart, page 331:

Important!!!

You should distinguish carefully between definite and indefinite integrals. A definite integral $\int_a^b f(x)\,dx$ is a *number*, whereas an indefinite integral $\int f(x)\,dx$ is a *function* (or family of functions). The connection between them is given by Part 2 of the Fundamental Theorem: if f is continuous on $[a, b]$, then

$$\int_a^b f(x)\,dx = \int f(x)\,dx\Big]_a^b$$

example

(a) $\int_{-4}^{4} \frac{1}{\sqrt{x}}\,dx = \text{DNE}$ b/c $\sqrt{x}$ is not defined over $[-4,4]$.

(b) $\int_1^4 \frac{1}{\sqrt{x}}\,dx = 2\sqrt{x}\Big|_1^4 = 2\sqrt{4} - 2\sqrt{1} = 2$

$$\int \frac{1}{\sqrt{x}}\,dx = \int x^{-1/2}\,dx = \frac{1}{-\frac{1}{2}+1}\,x^{-\frac{1}{2}+1} + C = 2x^{1/2} + C = 2\sqrt{x} + C$$

Anti-power rule for $\int x^p\,dx$ on next page.

Some basic indefinite integrals

- $\int x^p \, dx = \frac{1}{p+1} x^{p+1} + C$ if $p \neq -1$.

  Why? b/c $\frac{1}{p+1}$ is undefined when $p = -1$

- $\int \sin x \, dx = -\cos x + C$
- $\int \cos x \, dx = \sin x + C$
- $\int \sec^2(x) \, dx = \tan(x) + C$
- $\int \sec x \tan x \, dx = \sec x + C$

General Rule of Thumb!

Differentiation is easy.

Integration is hard.

(But both have lots of important applications.)

Anti-power Rule is true because:

$$\frac{d}{dx}\left[x^p\right] = p\,x^{p-1}, \quad p \text{ is a constant}$$

means $\int p\,x^{p-1}\,dx = x^p + C$

$$\| $$

$$p\int x^{p-1}\,dx$$

write $r = p-1$

Divide by p to get

$$\int x^{p-1}\,dx = \frac{1}{p}x^p + C$$

$p = r+1$

$$\int x^r\,dx = \frac{1}{r+1}x^{r+1} + C$$

$$\boxed{\int x^p\,dx = \frac{1}{p+1}x^{p+1} + C}$$

Property for Indefinite Integrals

$$\int a f(x) + b g(x)\,dx$$

$$= a\int f(x)\,dx + b\int g(x)\,dx$$

Linearity

Every rule for differentiation gives a rule for anti-differentiation, but these are not always very useful. For example:

Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

$$\int \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}\,dx = \frac{f(x)}{g(x)} + C$$

Complicated and not ~~very~~ useful.
at all

Working with Mathematical Variables

A pair of variables — say u and v — can be either "dependent" or "independent".

- The variables u and v are <u>independent</u> if changing one of the variables has no effect on the other variable. In other words each variable is considered to be constant with respect to the other.

- The variables u and v are <u>dependent</u> if changing the value of one does cause a change in the other variable. For example, if $v = f(u)$ for some function f then u and v are dependent. Another example would be if a and b are variables satisfying the equation $a^2 + b^2 = 1$ then a and b are dependent variables.

Important:

Basic Standing Assumption

If it is not explicitly stated that two variables are dependent then they are automatically considered to be independent.

Examples

$$= \int_{-3}^{0} x^2\,dx + \int_{-3}^{0} 1\,dx$$

① $$\int_{-3}^{0} x^2+1\,dx = \frac{1}{3}x^3 + x \Big|_{x=-3}^{0} = 0 - \left(\frac{1}{3}(-3)^3 + (-3)\right) = 12$$

$1 = x^0$

② $$\int_{-3}^{0} x^2+1\,dt = (x^2+1)t \Big|_{t=-3}^{0} = 3(x^2+1)$$

because x^2+1 is constant with respect to t.

③ If $x = 2t^3$ then

so x and t are dependent variables.

$$\int_{-3}^{0} x^2+1\,dt = \int_{-3}^{0} (2t^3)^2+1\,dt = \int_{-3}^{0} 4t^6+1\,dt$$

$$= \frac{4}{7}t^7 + t \Big|_{t=-3}^{0} = \frac{8769}{7}$$

④ $$\int x^2+1\,dt = (x^2+1)t + C$$

Conclusion: In either a definite integral $\int_a^b f(x)\,dx$ or an indefinite integral $\int f(x)\,dx$, the "differential" dx is a required and important part of the notation.

Strive to be meticulous in your use of mathematical notation ....