True or False?

① Any function with an absolute value in it is ~~scary~~ annoying.

~~True?~~ False

② $\int_2^4 |2x-3|\,dx = \int_2^4 2x-3\,dx$ True

$\int_2^4 |2x-3|\,dx = 6$, $\int_2^4 2x-3\,dx = 6$

$3/2$ is not in $[2,4]$

③ $\int_0^2 |2x-3|\,dx = \int_0^2 2x-3\,dx$ False

$\int_0^2 |2x-3|\,dx = 5/2$, $\int_0^2 2x-3\,dx = 25/4$

$\frac{3}{2}$ is in $[0,2]$

④ If you try to find the area of a region by computing an integral and get an answer of -10.5 then the area equals 10.5. False!

2 problems are possible:

- You set the integral incorrectly.
- You make an arithmetic/calculation mistake.

In either case you need to re-examine your work carefully!

Definition of absolute value.

$$|t| = \begin{cases} t & \text{if } t \geq 0 \\ -t & \text{if } t < 0 \end{cases}$$

E.G

$|10.5| = 10.5$ b/c $10.5 \geq 0$

$|-10.5| = -(-10.5) = 10.5$ b/c $-10.5 < 0$

$|0| = 0$

So

$$\boxed{|f(x)| = \begin{cases} f(x) & \text{if } f(x) \geq 0 \\ -f(x) & \text{if } f(x) < 0 \end{cases}}$$

example $\int_a^b |2x-3|\,dx = ?$

$$|2x-3| = \begin{cases} 2x-3 & \text{if } 2x-3 \geq 0 \\ -(2x-3) & \text{if } 2x-3 < 0 \end{cases}$$

Now ask: when is $2x-3 \geq 0$?

$$2x-3 \geq 0$$
add 3
$$2x \geq 3$$
divide by 2 *
$$x \geq 3/2$$

So $|2x-3| = \begin{cases} 2x-3 & \text{if } x \geq 3/2 \\ -2x+3 & \text{if } x < 3/2 \end{cases}$

more friendly than this

E.G.-

$$\int_0^4 |2x-3|\,dx = \int_0^{3/2} |2x-3|\,dx + \int_{3/2}^4 |2x-3|\,dx$$

$$= \int_0^{3/2} -2x+3\,dx + \int_{3/2}^4 2x-3\,dx$$

$$= \frac{9}{4} + \frac{25}{4} = 9$$

key: $3/2$ is inside $[0,4]$

<u>example</u> What does the graph of

$f(x) = |4x - x^2|$ look like?

degree 2 polynomial $\equiv$ quadratic polynomial

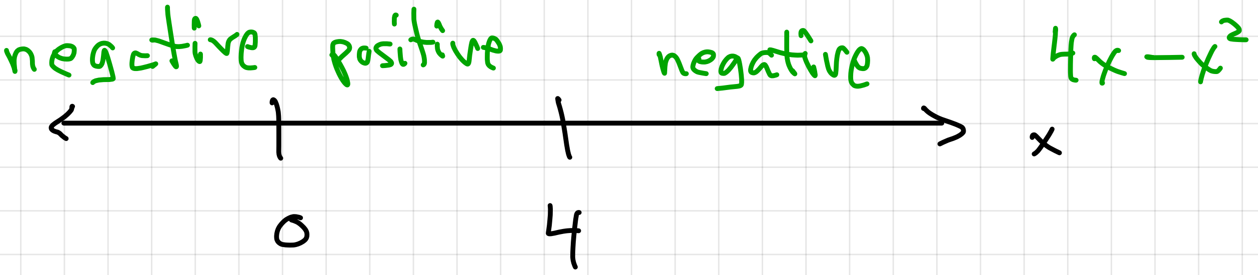
$$|4x-x^2| = \begin{cases} 4x - x^2 & \text{if } 4x - x^2 \geq 0 \\ -4x + x^2 & \text{if } 4x - x^2 < 0 \end{cases}$$

when is $4x - x^2 \geq 0$?

<u>First ask</u>: when is $4x - x^2 = 0$? $\Leftarrow$ $x = 0$ or 4

$$4x - x^2 = x(4-x) = 0$$

So $x = 0$ <u>or</u> $x = 4$ (when $4 - x = 0$)



<u>Conclusion</u>:

$$\underset{f(x)}{\underset{\parallel}{|4x - x^2|}} = \begin{cases} -4x + x^2 & \text{if } x < 0 \\ 4x - x^2 & \text{if } 0 \leq x \leq 4 \\ -4x + x^2 & \text{if } x > 0 \end{cases}$$

now describe graph:

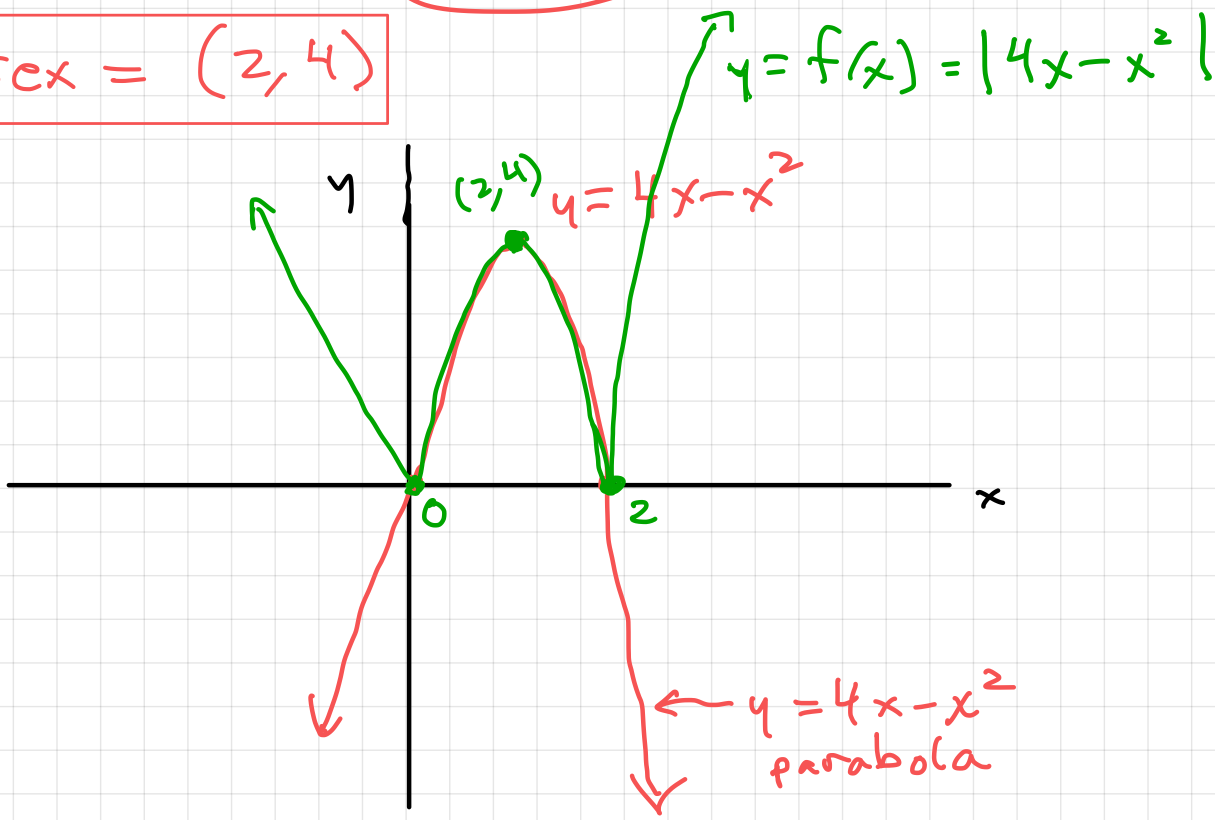
← goes thru (0, 0)

$y = 4x - x^2$ has graph that is a downward openward parabola. Where is its vertex?

use calculus $\qquad \dfrac{dy}{dx} = \dfrac{d}{dx}\left[4x - x^2\right] = 4 - 2x$

$\dfrac{dy}{dx} = 0$ when $x = 2$ ← x-value of vertex

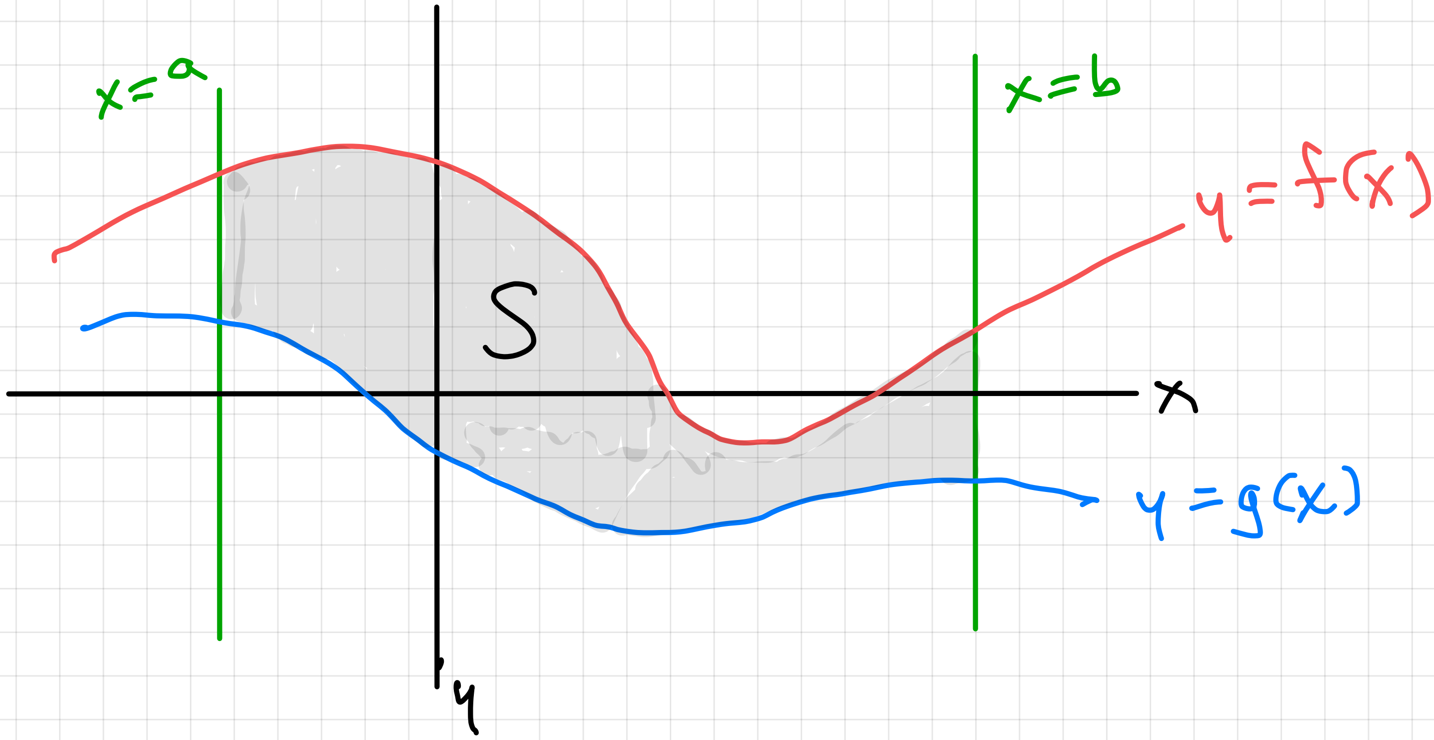
$\boxed{\text{So Vertex} = (2, 4)}$



(Take the mirror images in the x-axis of parts of the parabola $y = 4x - x^2$ below the x-axis.)

Areas ...

Type I Region

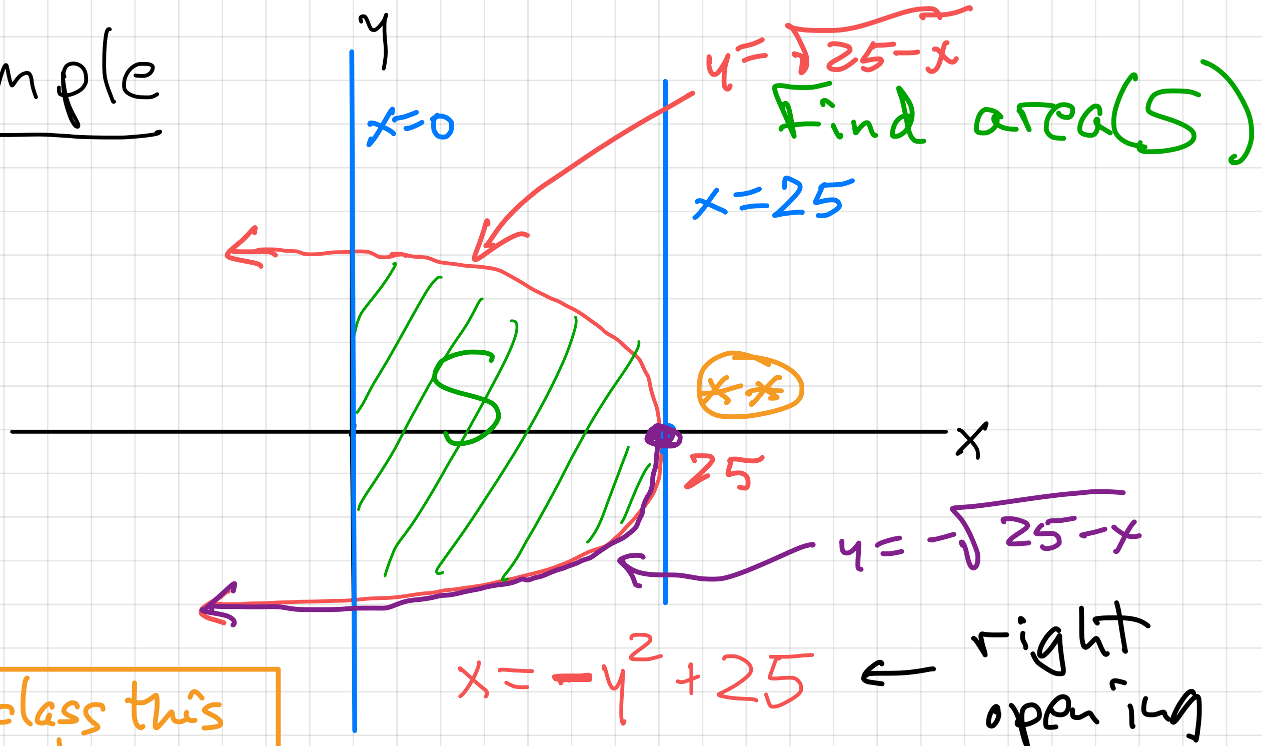


$$S: \begin{cases} a \le x \le b \\ g(x) \le y \le f(x) \end{cases}$$

$$\text{Area}(S) = \int_a^b f(x) - g(x)\, dx$$

example

Find area(S)



(**) In class this intercept was miscalculated!

Is S a type I region? Yes

$x=-y^2+25$ is not the graph of a function.

Try to solve for y:

$$-y^2 = x-25$$

$$y^2 = 25-x$$

take square root

$$y = \pm\sqrt{25-x}$$

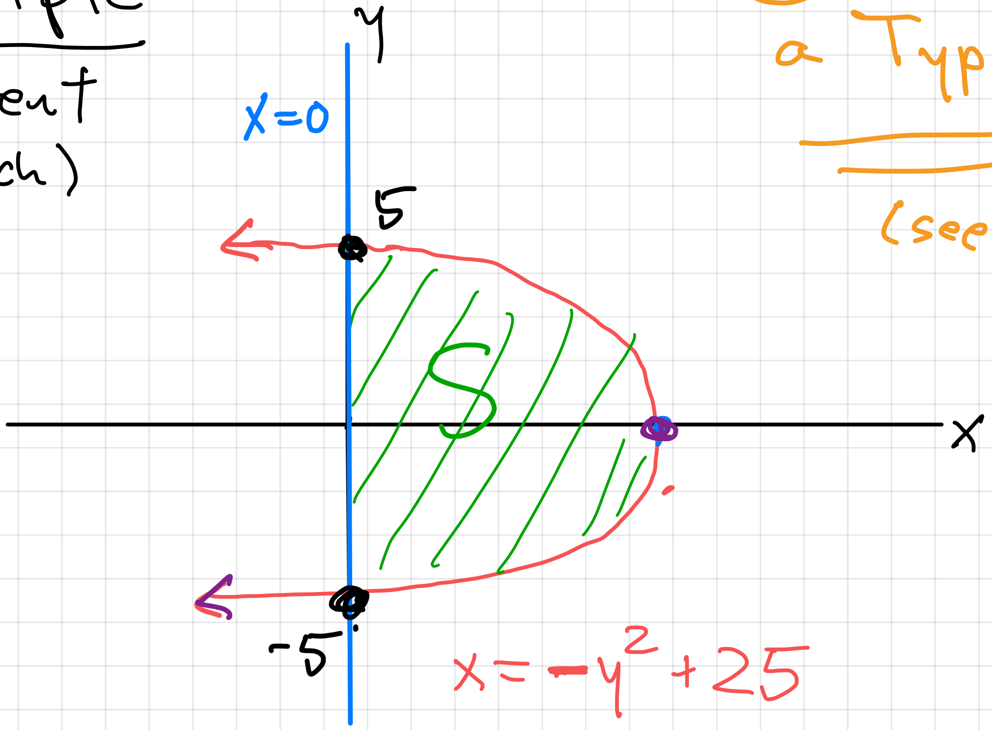
$$\begin{cases} u = 25-x \\ du = -dx \\ u(0)=25 \\ u(25)=0 \end{cases}$$

$$\text{Area}(S) = \int_0^{25} \sqrt{25-x} - \left(-\sqrt{25-x}\right) dx$$

$$= \int_0^{25} 2\sqrt{25-x}\, dx =$$

$$= \int_{25}^{0} -2u^{1/2}\, du = -2\frac{2}{3}u^{3/2}\Big|_{u=25}^{0} = \frac{500}{3}$$

same example
(different approach)



(*) We call this a Type II region

(see next page)

$$S = \begin{cases} -5 \le y \le 5 \\ 0 \le x \le -y^2 + 25 \end{cases}$$

$\leftarrow$ not Type I

but it is a type I region if x and y are interchanges. (*)

$$\text{Area}(S) = \int_{-5}^{5} (-y^2 + 25) - (0)\, dy$$

$$= \int_{-5}^{5} -y^2 + 25 \, dy$$

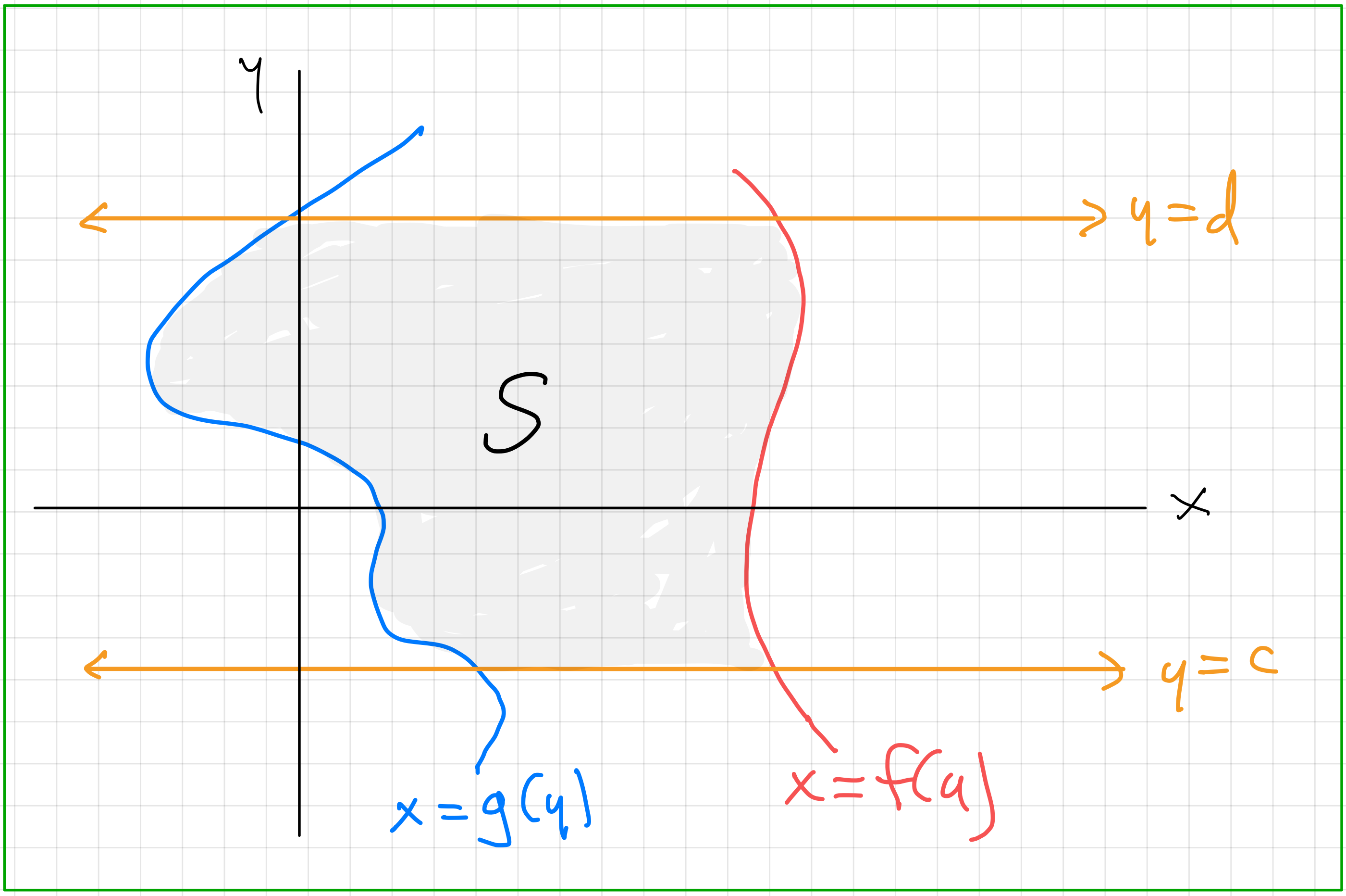
$$= -\frac{1}{3}y^3 + 25y \Big|_{-5}^{5} = 500/3$$

A Type II region S has form

bottom ↓   top ↓

$$S: \begin{cases} c \le y \le d \\ g(y) \le x \le f(y) \end{cases}$$

↑ left side   ↑ right side



(Horizontal lines on top and bottom. curves satisfying HLP on left and right.)

HLP $\equiv$ Horizontal Line Property