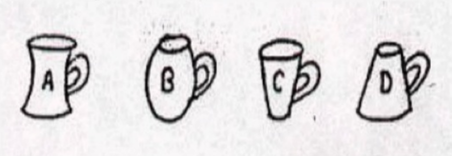
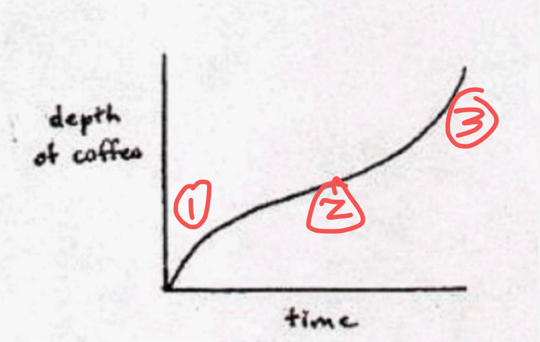
Here's a problem I found on WebWork.
It will not be on an assignment, but what do you think is the correct answer A, B, C, or D ??
Submit a guess on chat ?

(1 point) **Library/Rochester/setIntegrals20Volume/osu_in_20_7/osu_in_20_7.pg**



Coffee is poured into one of mugs above at a constant rate (constant volume per unit time). The graph below shows the depth of coffee in the mug as a function of time. (Click on images for better view.)



Which mug was filled with coffee? B

Be prepared to explain your choice (offline).

1 mug filling rapidly ~ mug is narrow
2 filling less rapidly ~ mug is wider
3 rapid ~ mug is narrower

Only mug B satisfies these properties.

Announcements:

1. Course syllabus is now posted at the website. Read this carefully when you have time. Don't hesitate to ask me any questions if you have them!

2. Short WebWork assignment is due this Friday by 11:59 PM.
   The next ones, wwork3 and wwork4, will be due next Wednesday and next Friday.

3. Attending class discussions sections is important and expected. Please contact me if this an issue for you.

Integrals and Riemann Sums (Review)

Given: a function $f(x)$, an interval $[a,b]$ and a positive integer N.

Form Riemann Sums:
subdivide $[a,b]$ into N subintervals with length $\Delta x = (b-a)/N$. Pick x_k^* in k^{th} subinterval
possibly in random fashion
form a sum

$$f(x_1^*)\Delta x + f(x_2^*)\Delta x + \cdots + f(x_N^*)\Delta x$$

$$= \sum_{k=1}^{N} f(x_k^*)\Delta x = R_N$$

Theorem If $f(x)$ is defined and continuous on $[a,b]$ then $\lim_{N\to\infty} R_N$ will exist and equal a finite number.

$$\int_a^b f(x)\,dx = \lim_{N\to\infty} R_N .$$

$\sum_i \to$ Sum $\to \int$

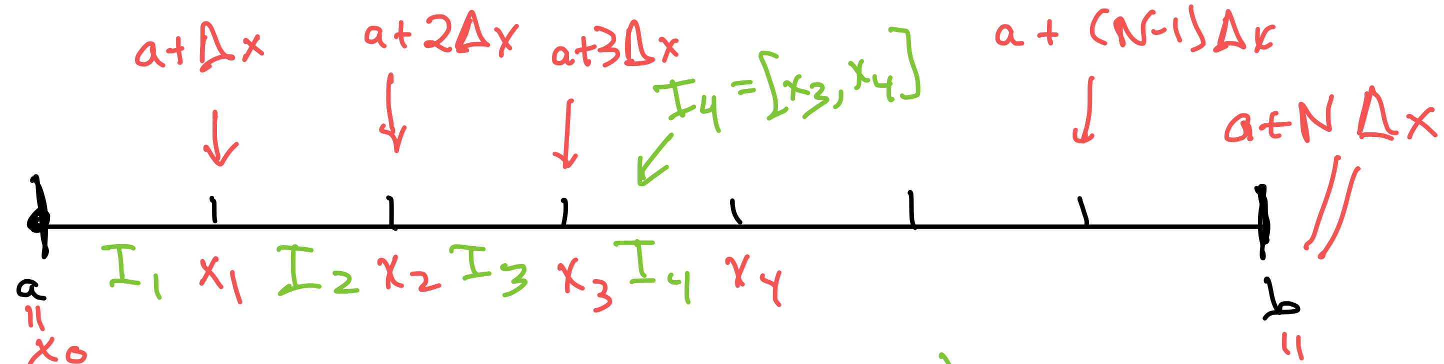
example

the integrand

$$\int_{-1}^{2} \frac{1}{x^3}\,dx = \text{DNE}$$

$\boxed{\text{here } [a,b]=[-1,2] \text{ but } \frac{1}{x^3} \text{ is not defined at } x=0.}$

Setting up a Riemann Sum using left- or right-endpoints: $\Delta x = (b-a)/N$



(This picture is for $N=7$.)

check: $a + N\Delta x = a + N\frac{b-a}{N} = a+b-a = b$

We write

$$x_0 = a,\ x_1 = a+\Delta x,\ x_2 = a+2\Delta x,\ \dots,\ x_N = b$$

so that the k^{th} subinterval is

$$I_k = [x_{k-1}, x_k]$$

← left endpoint of I_k is $x_{k-1} = a + (k-1)\Delta x$

where $k = 1, 2, \dots, N$.

So $x_{k-1} = a + (k-1)\Delta x$ = left-endpoint

$x_k = a + k\Delta x$ = right-endpoint

Left Riemann sum $\displaystyle\sum_{k=1}^{N} f(x_{k-1})\Delta x = R_N$

↑ choose $x_k^* = x_{k-1}$

right Riemann sum $\displaystyle\sum_{k=1}^{N} f(x_k)\Delta x = R_N$

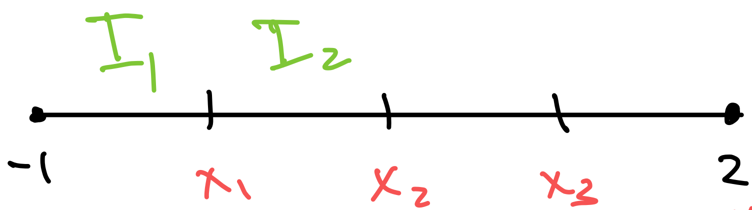
↑ choose $x_k^* = x_k$

Example $\int_{-1}^{2} x^3\,dx$

$f(x) = x^3$, $[a,b] = [-1,2]$, $N=4$

here:

$$\boxed{\begin{aligned}\Delta x &= \frac{b-a}{N} \\ &= \frac{2-(-1)}{4} = \frac{3}{4}\end{aligned}}$$



$x_0 = -1$

$x_1 = -1 + \frac{3}{4} = -1/4$

$x_2 = -1 + 2\frac{3}{4} = 1/2$

$x_3 = -1 + 3\frac{3}{4} = 5/4$

$x_4 = 2$

left Riemann Sum

$$R_4 = f(\overset{x_0}{-1})\frac{3}{4} + f(\overset{x_1}{-\tfrac{1}{4}})\frac{3}{4} + f(\overset{x_2}{\tfrac{1}{2}})\frac{3}{4} + f(\overset{x_3}{5/4})3/4$$

so

$$= (-1)^3\frac{3}{4} + \left(-\frac{1}{4}\right)^3\frac{3}{4} + \left(\frac{1}{2}\right)^3\frac{3}{4} + \left(\frac{5}{4}\right)^3\frac{3}{4}$$

$$= \frac{3}{4}\left((-1)^3 + \left(-\frac{1}{4}\right)^3 + \left(\frac{1}{2}\right)^3 + \left(\frac{5}{4}\right)^3\right)$$

right Riemann Sum

$$R_4 = \underset{x_1}{\left(-\frac{1}{4}\right)^3}\frac{3}{4} + \underset{x_2}{\left(\frac{1}{2}\right)^3}\frac{3}{4} + \left(\frac{5}{4}\right)^3\frac{3}{4} + \underset{x_4}{(2)^3}\frac{3}{4}$$

Basic Properties of Integrals

① $\int_a^b c\,dx = c(b-a)$ if c is constant

② $\int_a^b cf(x) + d\,g(x)\,dx$ if c, d are constants

$\shortparallel$

$c\int_a^b f(x)\,dx + d\int_a^b g(x)\,dx$

↑

This important property is called <u>linearity</u>.

③ $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx$

$= \int_a^c f(x)\,dx$

④ $\int_a^a f(x)\,dx = 0$