An **antiderivative** of $f(x)$ is a function $F(x)$ satisfying $F'(x) = f(x)$.

Section 3.9 of Stewart

## True/False Questions for chat (NFC)

FTTT

1. false $\cos(x)$ is an antiderivative of $\sin(x)$.
2. true $\sin(x)$ is an antiderivative of $\cos(x)$.
3. true An antiderivative of $\cos(x)$ is $\sin(x)-3$.
4. true There is one and only one antiderivative $F(x)$ of $\cos(x)$ with $F(0)=\sqrt{2}$.

see next page!

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(1) $\dfrac{d}{dx}\left[\cos x\right] = -\sin x$

antiderivative for $\sin x$ is $-\cos x$

(2) $\dfrac{d}{dx}\left[\sin x\right] = \cos x$

$\dfrac{d}{dx}\left[\sin(x) + C\right] = \cos x$

(4) $F(x) = \sin(x) + \sqrt{2}$

$F'(x) = \cos(x)$, $F(0) = \cancel{\sin(0)}^{\,0} + \sqrt{2}$

$= \sqrt{2}$

Stewart, page 278

**1 Theorem** If $F$ is an antiderivative of $f$ on an interval $I$, then the most general antiderivative of $f$ on $I$ is

$$F(x) + C$$

where $C$ is an arbitrary constant.

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Comment:

Every rule of differentiation gives a rule of antidifferentiation.

example : $\frac{d}{dx}[\sec x] = \sec x \tan x$

tells us that $\sec x$ is an antiderivative of $\sec x \tan x$.

So

what's an antiderivative of $\sec(x)$?

well, that's not so easy to answer. we won't work this out until Chapter 7.

# Announcements

- wwork3 and wwork4 due on Wednesday and Friday this week.
- Exam 1 scheduled for a week from this Friday.
- About discussion classes....
- Any questions about Course Syllabus?

# About Version 1 of FTC

$\int_a^b f(x)\,dx$ is a number called the integral of $f(x)$ over the interval $[a,b]$.

In the integral symbol the variable x is sometimes called a "dummy variable", because it doesn't have any bearing on the numerical value of the integral. In fact

$$\int_a^b f(x)\,dx = \int_a^b f(t)\,dt = \int_a^b f(\theta)\,d\theta = \int_a^b f(v)\,dv.$$

(However $\int_a^b f(x)\,dx \neq \int_a^b f(v)\,dx$ .... )

← will give examples later.

Observe that $\int_a^t f(x)\,dx$ is a number which depends on $t$, so it is a function of $t$.

$$\boxed{\text{FTC Version 1: } \quad \frac{d}{dt}\left[\int_a^t f(x)\,dx\right] = f(t).}$$

This could also be written as

$$\frac{d}{dt}\left[\int_a^t f(v)\,dv\right] = f(t)$$

$$\frac{d}{dx}\left[\int_a^x f(v)\,dv\right] = f(x)$$

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x) \qquad \text{etc...}$$

# Problems

① If $F(x) = \int_1^x t(t-2)\,dt$ when is $F(x)$ increasing / decreasing ?

FTC says $F'(x) = x(x-2)$

critical points for $F(x)$ at $x = 0$ and $x = 2$

$F' > 0$ | $F' < 0$ | $F' > 0$

$\longleftarrow$ F inc $\quad 0 \quad$ F dec $\quad 2 \quad$ F inc $\longrightarrow x$

② Find the derivative $\dfrac{d}{dx}\left[\int_0^{x^3+x} \sin(t)\,dt\right]$.

If $F(x) = \int_0^x \sin t\,dt$ then $F'(x) = \sin x$ by the FTC, and

$$F(x^3+x) = \int_0^{x^3+x} \sin(t)\,dt$$

Now use the chain rule:

$$\frac{d}{dx}\left[F(x^3+x)\right] = F'(x^3+x)\,\frac{d}{dx}\left[x^3+x\right]$$

$$= \sin(x^3+x)(3x^2+1)$$

**FTC Version 2**: If $F(x)$ is any antiderivative of $f(x)$ then

"F(x) evaluated from a to b."

$$\int_a^b f(x)\,dx = F(b) - F(a) = F(x)\Big|_a^b = F(x)\Big|_{x=a}^{b}$$

**Why?** FTC-version 1 says: $\int_a^t f(x)\,dx$ is an antiderivative of $f(t)$ as is $F(t)$. So

$F(t) = \int_a^t f(x)\,dx + C$. For some constant $C$

Then $F(b) - F(a) = \left(\int_a^b f(x)\,dx + C\right) - \left(\int_a^a \cancel{f(x)\,dx}^{\,0} + C\right)$

$= \int_a^b f(x)\,dx + C - C$

$= \int_a^b f(x)\,dx$

check $\frac{d}{dx}\left\{\frac{1}{2}x^2\right\} = \frac{1}{\cancel{2}}\cancel{2}x$

**examples**:

① $\int_{-1}^{1} x\,dx = \frac{1}{2}x^2\Big|_{-1}^{1} = \frac{1}{2}(1)^2 - \frac{1}{2}(-1)^2 = \frac{1}{2} - \frac{1}{2} = 0$

② $\int_0^{\pi/2} \sin\theta\,d\theta = -\cos\theta\Big|_0^{\pi/2} = -\cos(\pi/2) - (-\cos 0) = 1$

$= -(-1) = 1$

③ $\int_0^4 \sqrt{x}\,dx = \frac{2}{3}x^{3/2}\Big|_0^4 = \frac{2}{3}(4)^{3/2} - \frac{2}{3}(0)^{3/2}$

antiderivative is $\frac{2}{3}x^{3/2}$

$= \frac{2}{3}4^{3/2} = \frac{2}{3}\left(4^{1/2}\right)^3$

$= \frac{2}{3}2^3 = \frac{16}{3}$

# Notation for antiderivatives

no limits of integration!

Write $\int f(x)\,dx$ to denote the most general antiderivative of $f(x)$.

e.g. $\int x\,dx = \frac{1}{2}x^2 + C$

We call $\int f(x)\,dx$ the <u>indefinite integral of $f(x)$ with respect to $x$</u>

Stewart, page 331: Important!!!

You should distinguish carefully between definite and indefinite integrals. A definite integral $\int_a^b f(x)\,dx$ is a *number*, whereas an indefinite integral $\int f(x)\,dx$ is a *function* (or family of functions). The connection between them is given by Part 2 of the Fundamental Theorem: if $f$ is continuous on $[a, b]$, then

$$\int_a^b f(x)\,dx = \int f(x)\,dx\Big]_a^b$$

<u>example</u> $\int \sqrt{x}\,dx = \frac{2}{3}x^{3/2} + C = \frac{2}{3}x\sqrt{x} + C$

because $\frac{d}{dx}\left[\frac{2}{3}x^{3/2}\right] = \frac{2}{3}\cdot\frac{3}{2}x^{3/2-1} = x^{1/2} = \sqrt{x}$

# Some basic indefinite integrals

- $$\int x^p \, dx = \frac{1}{p+1} x^{p+1} + C \quad \text{if } p \neq -1.$$
- $$\int \sin x \, dx = -\cos x + C$$
- $$\int \cos x \, dx = \sin x + C$$
- $$\int \sec^2(x) \, dx = \tan(x) + C$$
- $$\int \sec x \tan x \, dx = \sec x + C$$

## General Rule of Thumb!

Differentiation is easy.

Integration is hard.

(But both have lots of important applications.)