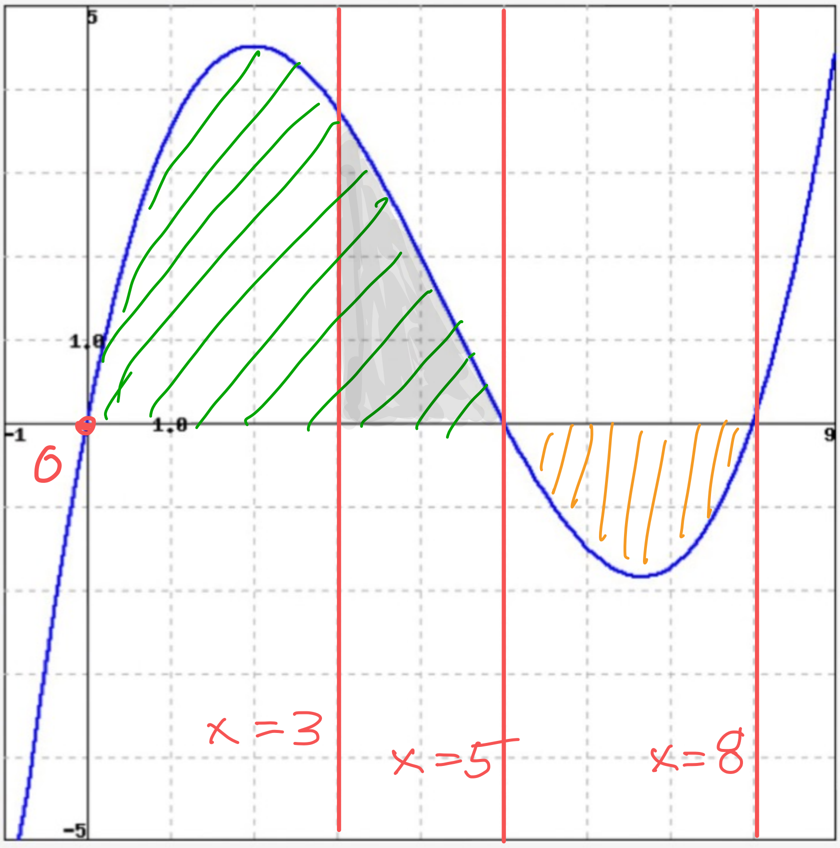
Can you solve this WebWork problem?

For the function f whose graph is given below, list the following quantities in increasing order, from smallest to largest.

A: $\int_0^8 f(x)\,dx$ B: $\int_0^3 f(x)\,dx$ C: $\int_5^8 f(x)\,dx$ D: $\int_0^5 f(x)\,dx$



$$\int_0^5 f(x)\,dx = \int_0^5 f(x) - 0\,dx = \text{area of green shaded region}$$

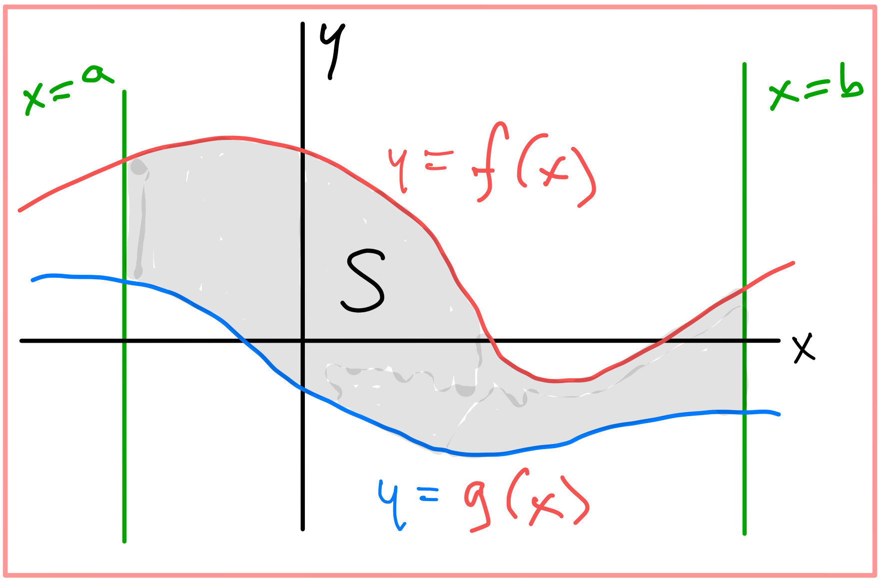
$$\text{area of orange shaded region} = \int_5^8 0 - f(x)\,dx = -\int_5^8 f(x)\,dx$$

$$\underset{C}{\int_5^8 f(x)\,dx} < \underset{B}{\int_0^3 f(x)\,dx} \overset{*}{<} \underset{A}{\int_0^8 f(x)\,dx} < \underset{D}{\int_0^5 f(x)\,dx}$$

* looks like area(gray region) > area(orange region)

A Type I region has inequalities:

$$S: \begin{cases} \overset{\text{left}}{a} \le x \le \overset{\text{right}}{b} \\ \underset{\text{bottom}}{g(x)} \le y \le \underset{\text{top}}{f(x)} \end{cases}$$



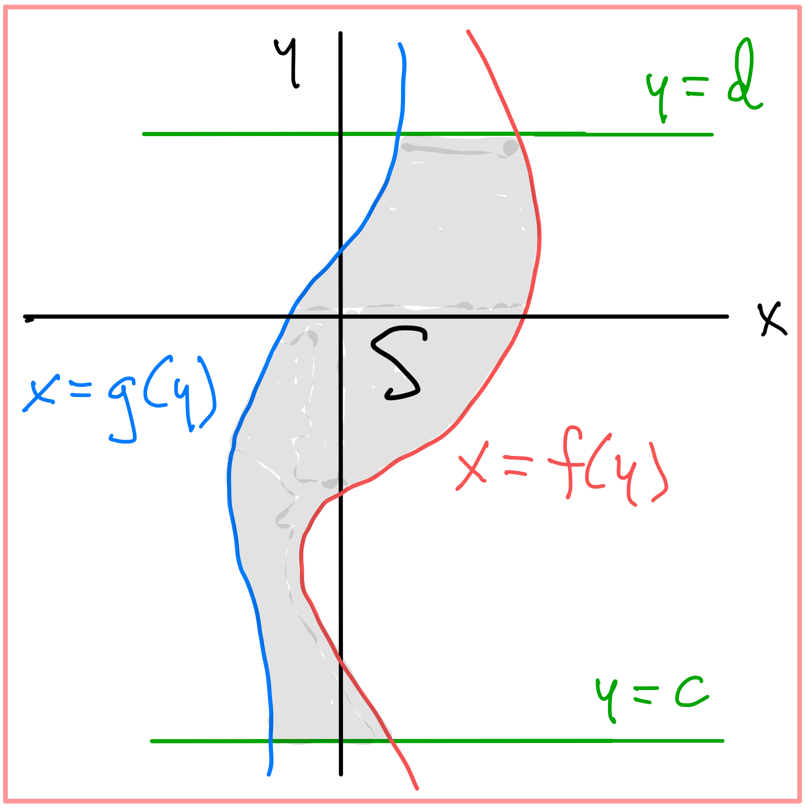
$$\text{Area}(S) = \int_a^b \underset{\text{top}}{f(x)} - \underset{\text{bottom}}{g(x)}\,dx$$

Interchanging the roles of x and y gives:

Type II region

$$S: \begin{cases} \overset{\text{bottom}}{c} \le y \le \overset{\text{top}}{d} \\ \underset{\text{left}}{g(y)} \le x \le \underset{\text{right}}{f(y)} \end{cases}$$

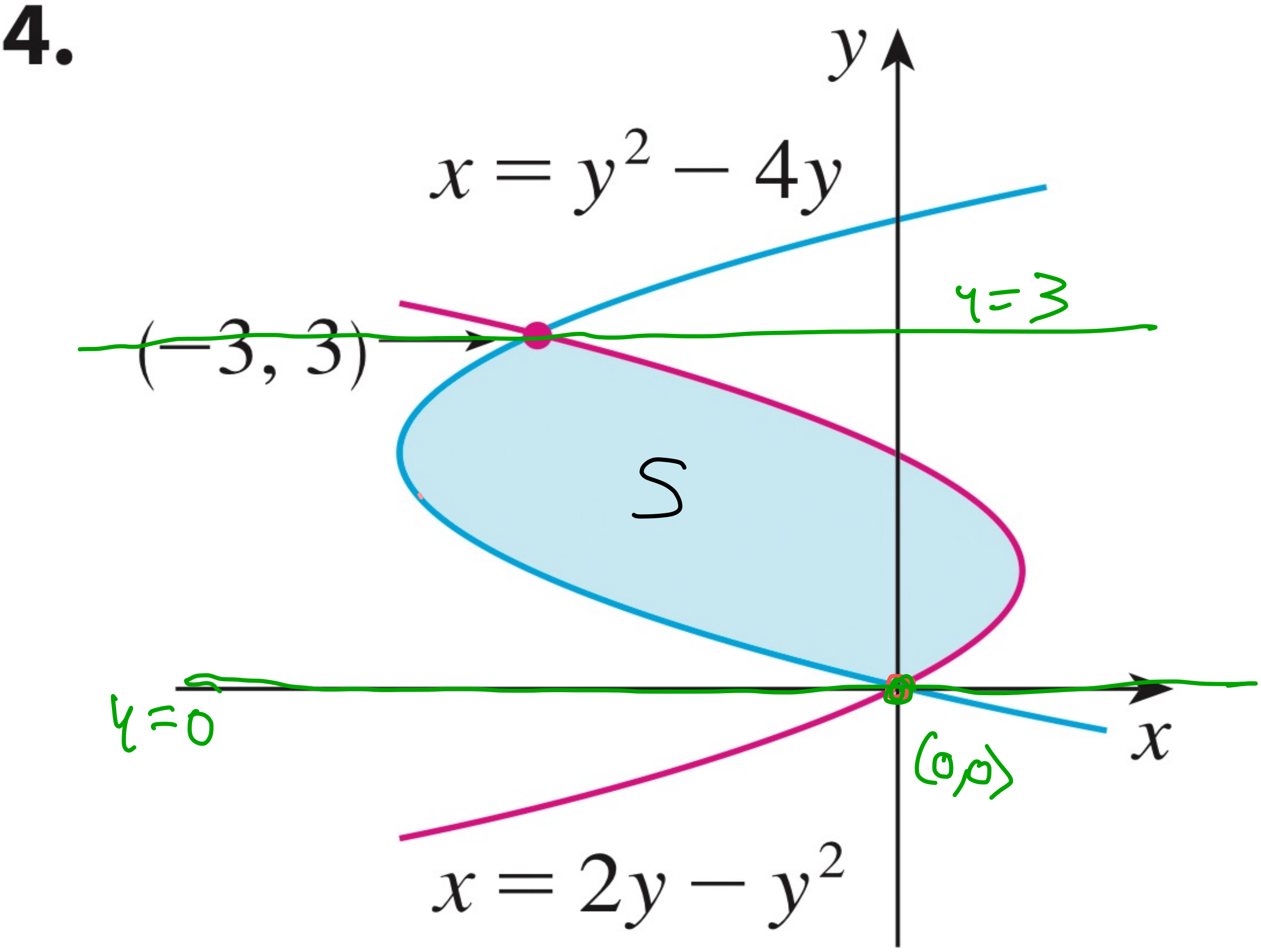
$$\text{Area}(S) = \int_c^d f(y) - g(y)\,dy$$



note: The graph of a function $x=g(y)$ satisfies:

HLP: Each horizontal line intersects the curve in at most one point.

Stewart exercise, p. 362. Find the area.



Type II region

The region is $S: \begin{cases} \overset{c}{\overset{\shortparallel}{0}} \le y \le \overset{d}{\overset{\shortparallel}{3}} \\ \underbrace{y^2-4y}_{g(y)} \le x \le \underbrace{2y-y^2}_{f(y)} \end{cases}$

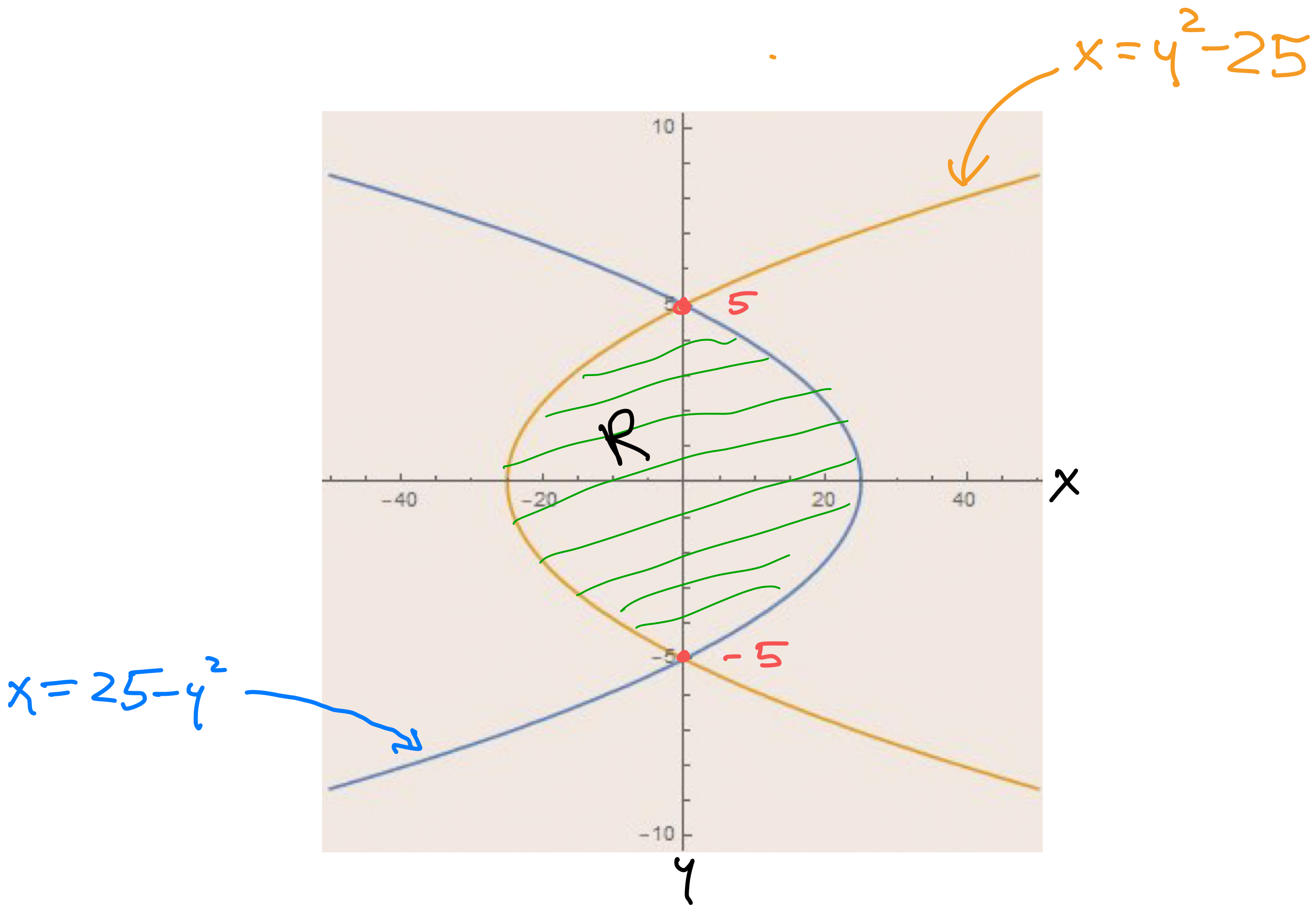
$$\text{Area}(S) = \int_0^3 (2y-y^2) - (y^2-4y)\, dy = \int_0^3 -2y^2 + 6y\, dy$$

$$= -\frac{2}{3}y^3 + 3y^2 \Big|_{y=0}^{3} = -18 + 27 = 9$$

To analyze S as a type I region it would be necessary to break S into 3 pieces with the lines $x=-3$ and $x=0$.

Example: Find the area of the region $\underset{R}{\text{between}}$

$x = 25 - y^2$ and $x = y^2 - 25$.



The region R can be described by inequalities:

$$R: \begin{cases} -5 \le y \le 5 \\ y^2 - 25 \le x \le 25 - y^2 \end{cases}$$

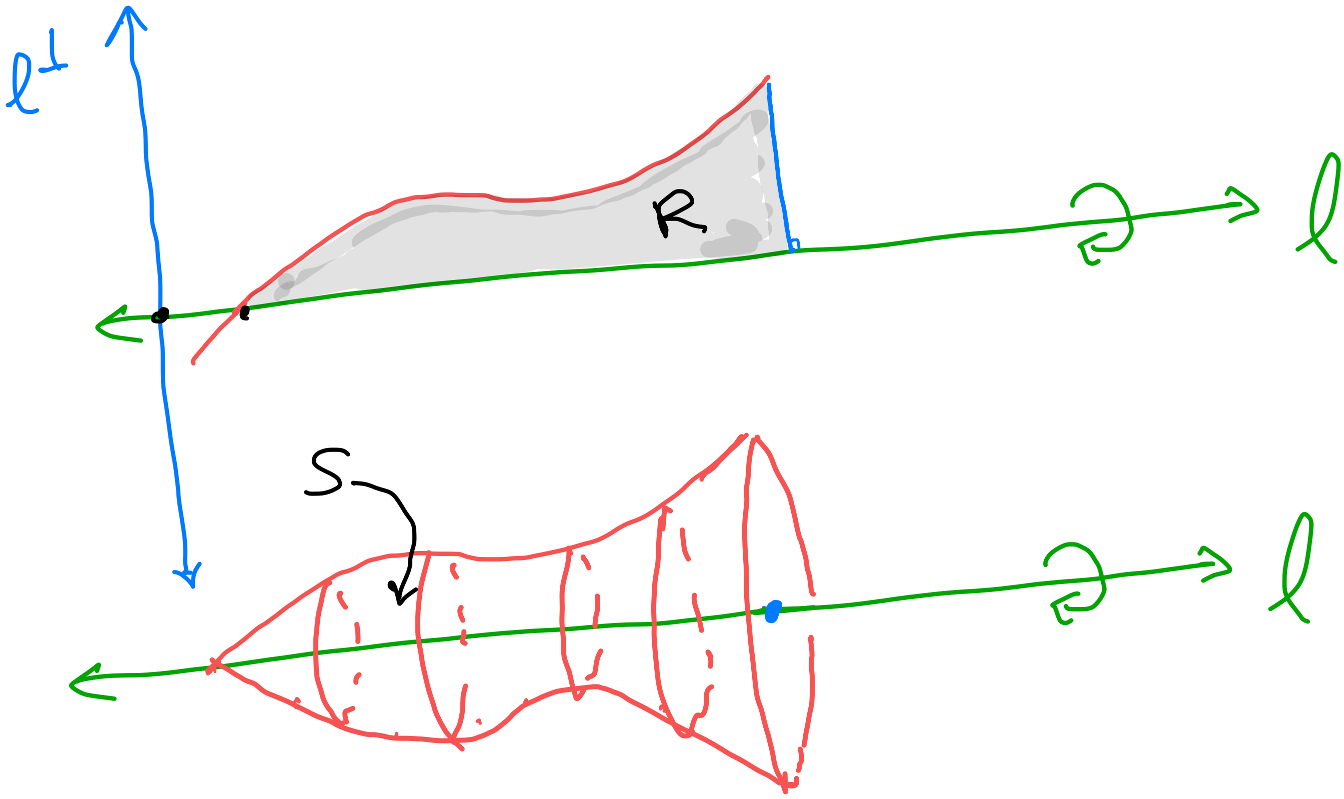
This is a Type II region.

$$\text{Area}(R) = \int_{-5}^{5} (25 - y^2) - (y^2 - 25)\, dy = \int_{-5}^{5} 50 - 2y^2\, dy = 50y - \frac{2}{3}y^3 \Big|_{-5}^{5}$$

$$= \left(250 - \frac{250}{3}\right) - \left(-250 + \frac{250}{3}\right) = 500 - \frac{500}{3} = 1000/3$$

VOLUME (sections 5.2 and 5.3)

A solid of revolution is constructed from a planar region R and a line ℓ in the same plane by rotating R around ℓ. The resulting solid S has ℓ as a "rotational axis of symmetry".



We will discuss two methods for finding the volume of S.

① **Disk or Washer method:**

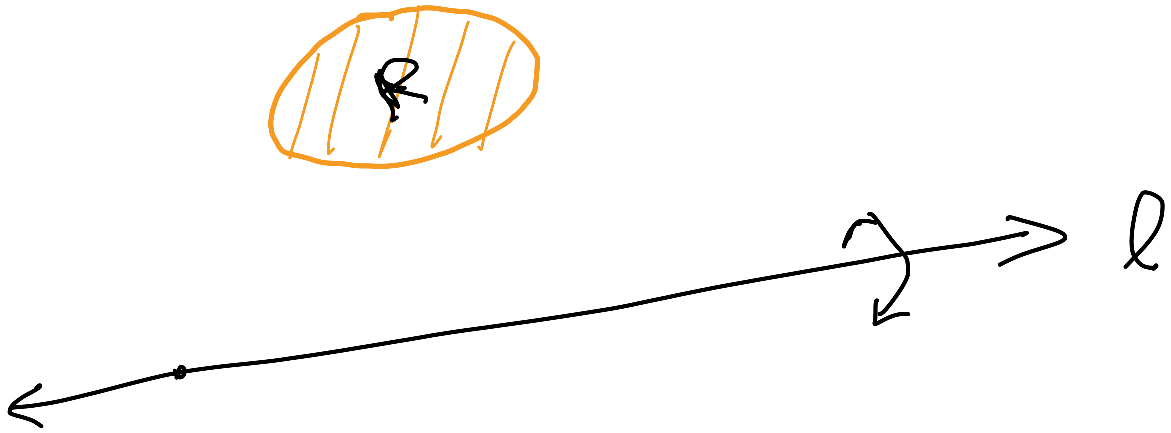
use ℓ as reference line.

② **Cylindrical shell method:**

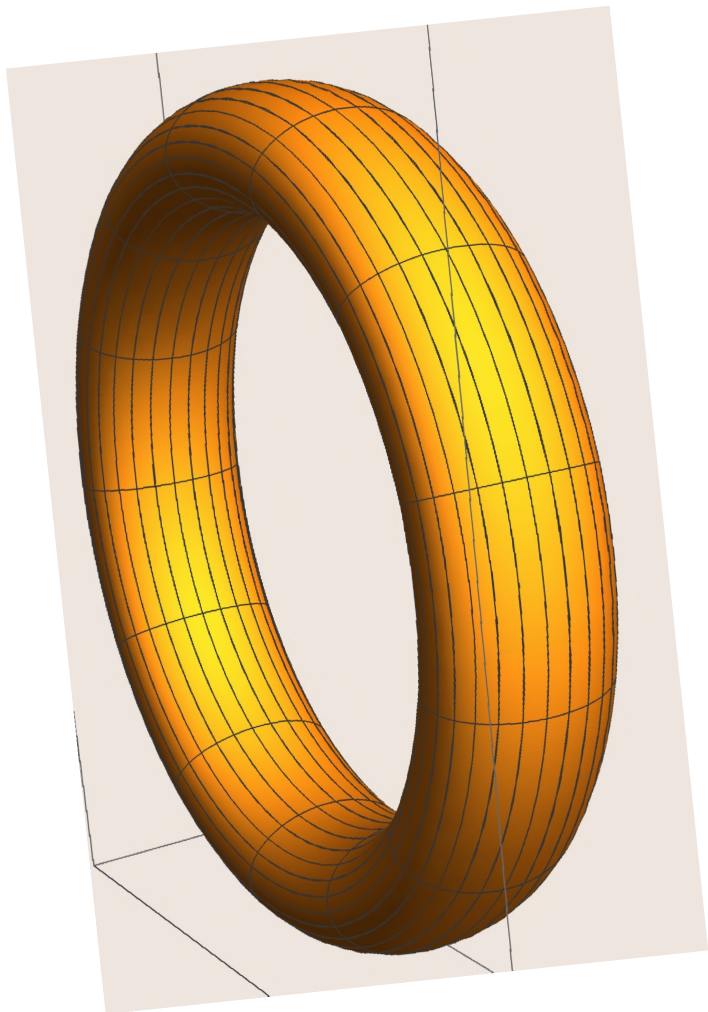
use a line $\ell^\perp$ perpendicular to ℓ as reference line.

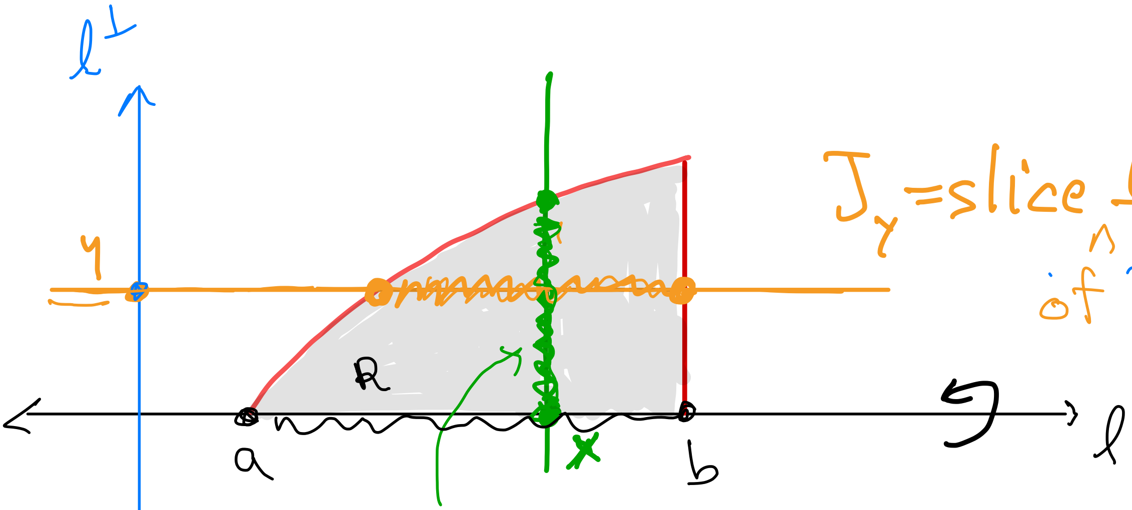
$\perp \equiv$ "perpendicular"

example:



The resulting solid of revolution is doughnut shaped:

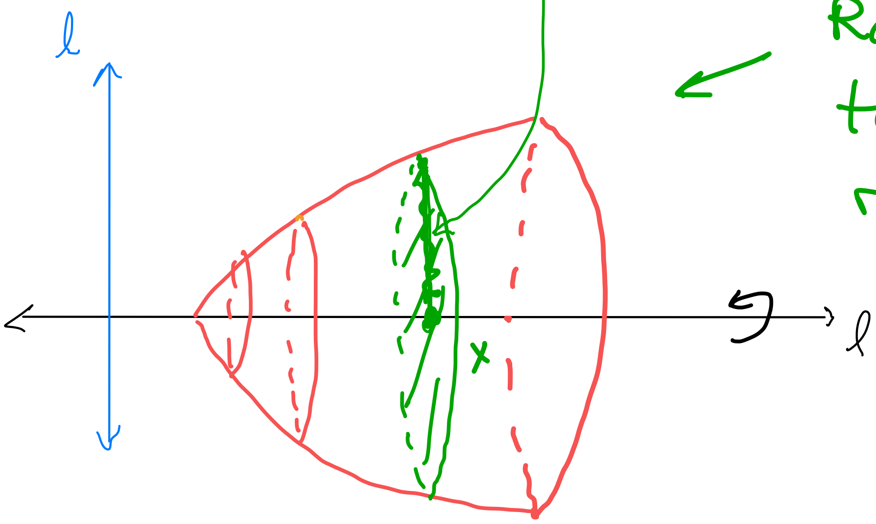




$J_y = $ slice $\perp\ \ell^\perp$ of R

$I_x = $ slice $\perp\ \ell$ at x of R

washer method

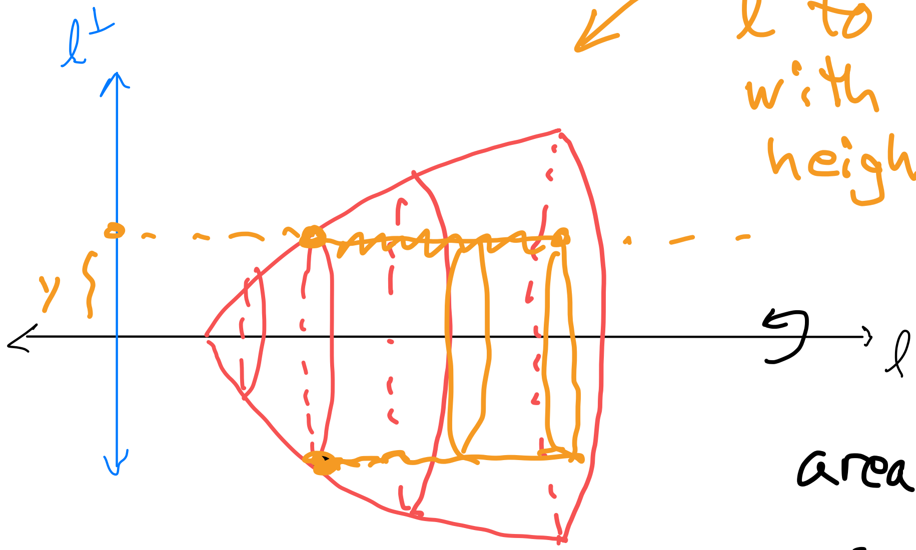


Rotate I_x around ℓ to get a disk of radius $\mathrm{lgth}(I_x)$

area(disk)

$= \pi\, \mathrm{lgth}(I_x)^2$

shell method



Rotate J_y around ℓ to get a cylinder with radius y and height $\mathrm{lgth}(J_y)$.

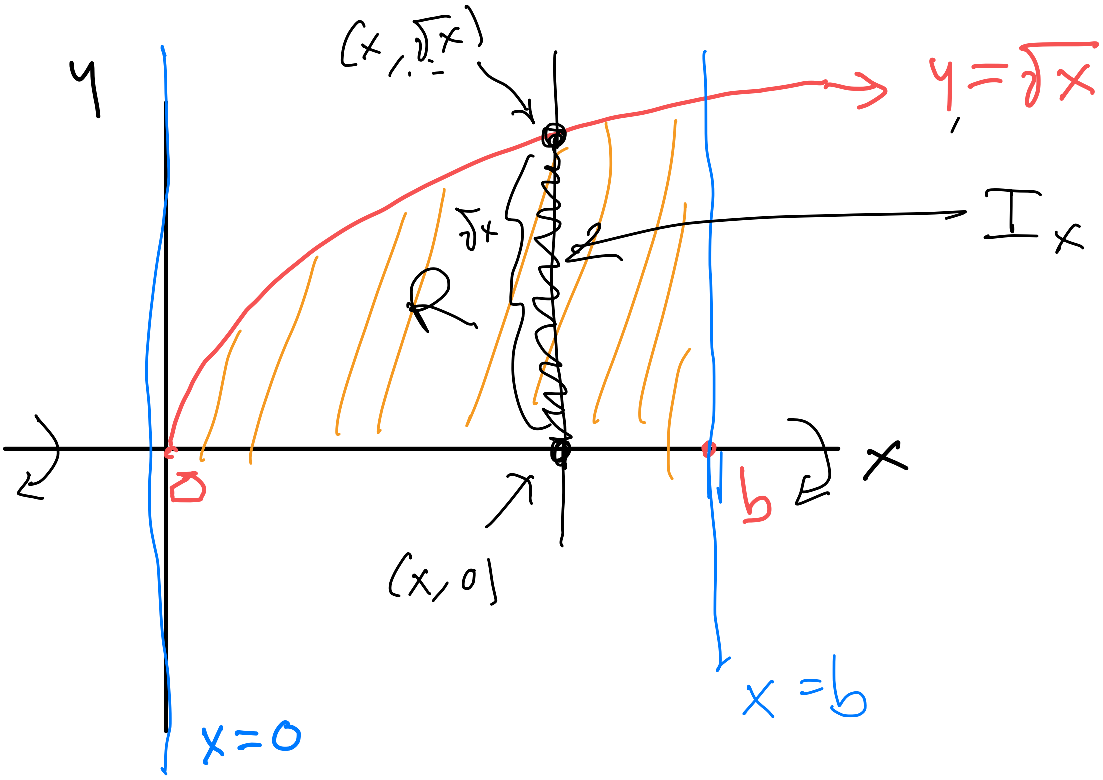
area(cylinder)

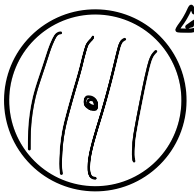
$= 2\pi y\, \mathrm{lgth}(J_y)$

Disk Method - section 5.2

$$\text{Volume } S = \int_a^b \pi \, \mathrm{lgth}(I_x)^2 \, dx$$

Example: Find the volume of the solid obtained by rotating the region below $y=\sqrt{x}$ in Quadrant I between $x=0$ and $x=b$ around x-axis





rotate I_x around x-axis to get a disk of radius $\sqrt{x}$.

$$\text{Volume}(S) = \int_0^b \pi (\sqrt{x})^2 \, dx$$

$$= \int_0^b \pi x \, dx = \frac{\pi}{2} x^2 \Big|_0^b = \frac{\pi b^2}{2}$$